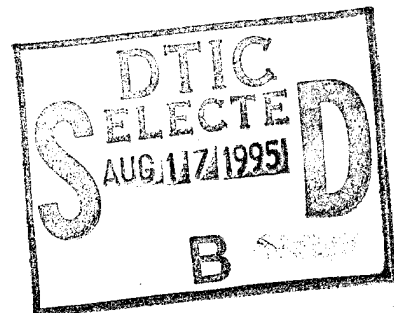


# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



## THESIS



A TIME DOMAIN APPROACH TO SENSITIVITY  
ANALYSIS OF DIRECT DETECTION OPTICAL  
FDMA NETWORKS WITH OOK MODULATION

by

John Anthony Studer

March 1995

Thesis Advisor:

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# A TIME DOMAIN APPROACH TO SENSITIVITY ANALYSIS OF DIRECT DETECTION OPTICAL FDMA NETWORKS WITH OOK MODULATION

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## ABSTRACT

We derive the closed form expression for the probability of bit error in direct detection, dense wavelength division multiplexed (WDM) fiber optic systems employing OOK as the modulation technique and single-cavity Fabry-Perot (FP) filters in the receiver as demultiplexers. The expression is derived in the time domain using the impulse response of the single-cavity Fabry-Perot filter and the complex baseband equivalent of the received dense WDM optical signal. The two are convolved to produce the FP filtered output signal  $s(t)$ . We then integrate  $\mathcal{R}|s(t)|^2$  over one bit period, where  $\mathcal{R}$  is the responsivity (A/W) of the photodetector following the FP filter in the receiver structure. This integral is the deterministic  $X$  of the decision variable  $Y$  where  $Y = X + N$ .  $N$  is the postdetection thermal noise (amplifier generated), a zero mean Gaussian random variable with variance  $N_0T$  where  $N_0$  is the noise current spectral density ( $A^2/Hz$ ). Both  $X$  and  $N$  are combined into an expression for probability of bit error. A limited case of the complete model is assumed, and probability of bit error graphs are generated.



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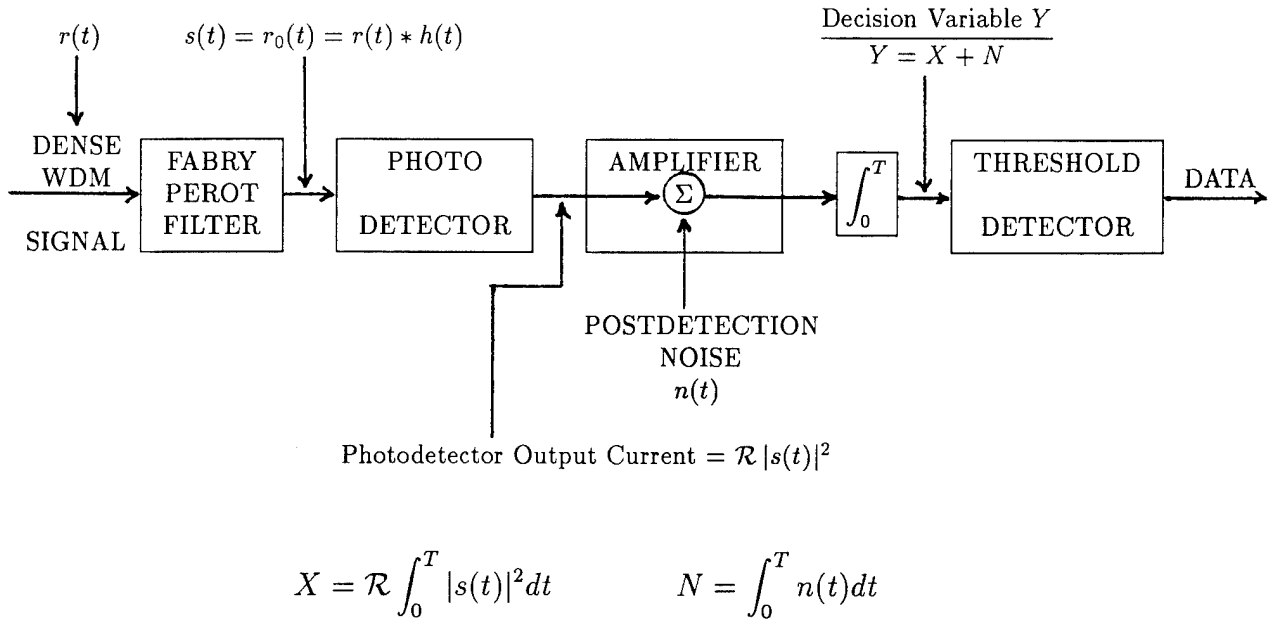
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# I. INTRODUCTION

Direct detection optical frequency division multiple access (FDMA) networks are increasingly becoming an attractive alternative to coherent optical FDMA networks [1]. One of the primary reasons is that noncoherent systems do not require expensive synchronization circuitry for proper operation. Also, present optical filter technology allows designers to closely pack the channels in frequency, resulting in dense wavelength division multiplexed (WDM) systems that can provide aggregate bit rates of many terabits per second ( $1\text{T b/sec} = 10^{12}\text{ bits/sec}$ ) [2]. The possible uses of dense WDM systems are many (local area networks, undersea surveillance, etc.), but one can easily see the enormous economic benefit and importance of being able to transmit aggregate bit rates of terabits per second on a single fiber without the economic burden of expensive synchronization circuitry imposed by a coherent system. In this thesis, we derive the complete closed form expression in the time domain for the probability of bit error of dense WDM systems employing OOK as the modulation technique and single-cavity Fabry-Perot (FP) filters as channel demultiplexers. In our derivation, we make no simplifying mathematical assumptions and use the impulse response of the single-cavity Fabry-Perot filter, which is an infinite sum of delayed impulses whose intensities decrease geometrically. The results of our work are presented in Chapters II, III, and IV. A detailed derivation of the deterministic signal component of the decision variable appearing at the output of the integrator of the Channel 0 (Channel of Interest) receiver appears in Appendix A. See Fig. 1 for a sketch of this receiver. Appendix B shows how the complete model is reduced for a limited case and how probability of bit error calculations are made for this limited case. Appendix C shows the strategy and programs used to obtain the probability of bit error for four values of free spectral range-bit period product

and a given range of signal-to-noise ratios. It is interesting to note at this point that, although the model presented in this thesis is strictly derived, complete, and done without approximation, its major weakness is that it is computationally very intensive. In fact, it takes several months of computer time on several SPARC-10 workstations working simultaneously to generate a single graph of the probability of bit error for this system. At the end of the thesis we conclude that, although our model is mathematically correct, a discrete time approach to the problem is probably a more efficient investigative tool in analyzing the performance of dense WDM fiber optic networks.



**Figure 1:** Channel 0 OOK receiver structure.

## II. ANALYSIS

An OOK, dense WDM system utilizing single-cavity Fabry-Perot (FP) optical filters as demultiplexers consists of  $M + 1$  transmitters (fixed wavelength lasers) connected over a fiber link to  $M + 1$  receivers. Each receiver contains a frequency selective Fabry-Perot filter to demultiplex one of the  $M + 1$  channels. In our derivation of the closed form expression for the probability of bit error for this system,  $M$  is the even number of adjacent channels symmetrically placed in frequency around the Channel of Interest (Channel 0) whose carrier is transmitted on an arbitrary wavelength  $\lambda_a$ . The Channel 0 receiver is shown in Fig. 1. The dense WDM optical signal is received by the Fabry-Perot filter tuned to  $\lambda_a$ , which allows the Channel 0 data signal to pass and rejects signals in the adjacent channels. A photodetector then converts the filtered optical signal to a current. The photodetector has an arbitrary responsivity  $\mathcal{R}$  (A/W). The current is then amplified by a low-noise amplifier which adds a postdetection thermal noise  $n(t)$  with two-sided current spectral density  $N_0$  ( $A^2/Hz$ ). Both the signal and noise currents are now passed to an integrator (low pass filter). The output of the integrator is the decision variable  $Y$ , which is compared to a threshold  $V_T$  to determine whether an "0" or "1" was sent.

### A. THE DENSE WDM SIGNAL AT THE INPUT OF THE FABRY-PEROT FILTER: $r(t)$

Again we note that Channel 0 is our channel of interest. We can write the expression for the Channel 0 data signal in the complex baseband as

$$b_0(t) = \sum_{i=-L_0}^0 b_{0,i} p_T(t - iT) \quad (1)$$

where  $T$  is the data bit period (s), and  $b_{0,i}$  is the bit in Channel 0 during the time period  $[iT, (i + 1)T]$ . Note that  $b_{0,i} \in \{0, 1\}$ .  $L_0$  is a positive integer which represents

the number of bits in Channel 0 that are trailing the detected bit or bit of interest  $b_{0,0}$ .  $p_T(t)$  is the rectangular pulse function defined as

$$p_T(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

Channel  $k$  is any arbitrary adjacent channel. There are  $M$  adjacent channels placed symmetrically in frequency around Channel 0.  $M$  is an even integer. The indices for  $k$  are as follows  $k = -M/2, \dots, -1, 1, \dots, +M/2$ . We may now write the complex baseband expression for the  $k^{\text{th}}$  channel data signal

$$b_k(t) = \sum_{\ell=-L}^0 b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \quad (3)$$

where  $\omega_k$  is the radian frequency spacing between Channel 0 and Channel  $k$  and  $\omega_k = -\omega_{-k}$ . We have already defined the pulse function  $p_T(t)$  in Eq. (2) above.  $L$  is a positive integer which represents the number of bits in Channel  $k$  which trail the bit in Channel  $k$  that is the  $0^{\text{th}}$  bit,  $b_{k,0}$ . We now note that  $b_{k,\ell}$  is the bit in Channel  $k$  during the time period  $[\ell T, (\ell + 1)T]$  and that  $b_{k,\ell} \in \{0, e^{j\phi_k}\}$ , where  $\phi_k$  is Channel  $k$ 's phase offset from Channel 0.  $\phi_k$  is assumed to be a uniformly distributed random variable between 0 and  $2\pi$  ( $\phi_k \sim U[0, 2\pi]$ ). We can now use Eqs. (1) and (3) to write the complex baseband equivalent dense WDM received optical signal which appears at the input of the Channel 0 Fabry-Perot filter (see Fig. 1)

$$r(t) = \sqrt{P} b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sqrt{P} b_k(t) \quad (4)$$

and

$$r(t) = \sqrt{P} \left( \sum_{i=-L_0}^0 b_{0,i} p_T(t - iT) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \right) \quad (5)$$

where  $P$  is the received optical power.

## B. THE OPTICAL OUTPUT OF THE FABRY-PEROT FILTER:

$$s(t) = r_0(t)$$

To arrive at the optical output of the Fabry-Perot filter  $s(t)$ , we must convolve the input signal  $r(t)$  with the impulse response  $h(t)$  of the filter

$$s(t) = r_0(t) = r(t) * h(t) = h(t) * r(t) \quad (6)$$

First, however, we need the expression for  $h(t)$ . The Fabry-Perot filter is a causal, linear, time-invariant (LTI) system. It can be shown [1] that

$$h(t) = (1 - \rho) \sum_{g=0}^{\infty} \rho^g \delta\left(t - \frac{g}{\beta}\right) \quad (7)$$

where  $\delta(t)$  is the *Dirac Delta Function* defined in two parts

$$\delta(t) = \begin{cases} 0, & t < 0 \\ 0, & t > 0 \end{cases} \quad (8a)$$

and

$$\int_{0^-}^{0^+} \delta(t) dt = 1 \quad (8b)$$

Looking at Eq. (7), we note that  $\rho$  is the power reflectivity of the single-cavity Fabry-Perot filter and  $\beta$  is the filter's free spectral range (Hz). Performing the convolution operation yields the following (see Appendix A for details)

$$\begin{aligned} s(t) = r_0(t) = & \underbrace{\sqrt{P}(1 - \rho)}_K \left[ \underbrace{\sum_{g=0}^{\infty} \rho^g b_{0,0} p_T \left(t - \frac{g}{\beta}\right)}_{s_B(t)} \right. \\ & + \underbrace{\sum_{i=-L_0}^{-1} \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left(t - \frac{g}{\beta} - iT\right)}_{s_{ISI}(t)} \\ & + \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left(t - \frac{g}{\beta} - \ell T\right)}_{s_{ACI}(t)} \left. \right] \quad (9) \end{aligned}$$

where  $s_B(t)$  is the desired signal (signal of interest),  $s_{ISI}(t)$  is the intersymbol interference signal, and  $s_{ACI}(t)$  is the adjacent channel interference signal. Then

$$s(t) = r_0(t) = K (s_B(t) + s_{ISI}(t) + s_{ACI}(t)) \quad (10)$$

We are now interested in writing an expression for  $s(t)$  during the detection interval  $0 \leq t \leq T$ . We do this by sketching  $s_B(t)$  from Eq. (9) and then writing another expression which accounts for how the rectangular pulses behave during the detection interval. We then sketch the pulse functions for the first three values of  $i$  for  $s_{ISI}(t)$  in Eq. (9). We note how they appear during the detection interval, and recognizing a pattern, we write another expression for  $s_{ISI}(t)$  for  $0 \leq t \leq T$ . A similar process is followed to arrive at  $s_{ACI}(t)$  for  $0 \leq t \leq T$ . We now present  $s_B(t)$ ,  $s_{ISI}(t)$ , and  $s_{ACI}(t)$  for  $0 \leq t \leq T$  (see Appendix A for details)

$$s_B(t) = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right), \quad \text{for } 0 \leq t \leq T \quad (11)$$

where  $q = T/(1/\beta) = \beta T$  is an integer. Recall that  $T$  is the data bit period (s), and  $\beta$  is the single-cavity Fabry-Perot filter's free spectral range (Hz).

$$s_{ISI}(t) = \sum_{i=-L_0}^{-1} \left[ \sum_{g=-(1+i)q+1}^{(-iq)-1} \rho^g b_{0,i} p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) + \sum_{g=-iq}^{-(i-1)q-1} \rho^g b_{0,i} p_T \left( \frac{T}{\left(T - \left(\frac{g+iq}{\beta}\right)} \left(t - \left(\frac{g+iq}{\beta}\right)\right) \right) \right] \right], \quad \text{for } 0 \leq t \leq T \quad (12)$$



$$\begin{aligned}
s_{ACI}(t) = & \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \left( \left[ \sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right] \right. \\
& + \sum_{\ell=-L}^{-1} \left[ \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{\beta T(t)}{g + (1+\ell)q} \right) \right. \\
& + \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} \times \dots \\
& \left. \left. p_T \left( \frac{T}{\left(T - \left(\frac{g+\ell q}{\beta}\right)} \right) \left(t - \left(\frac{g+\ell q}{\beta}\right)\right) \right) \right] \right], \quad \text{for } 0 \leq t \leq T \quad (13)
\end{aligned}$$

Recall Eqs. (9) and (10) where

$$s(t) = K(s_B(t) + s_{ISI}(t) + s_{ACI}(t))$$

and

$$K = \sqrt{P}(1 - \rho)$$

We can see that  $s(t)$  in the interval  $0 \leq t \leq T$  can now be calculated by substituting Eqs. (11), (12), and (13) into their appropriate positions in Eq. (10).

### C. COMPUTATION OF $X = \mathcal{R} \int_0^T |s(t)|^2 dt$ , THE SIGNAL COMPONENT OF THE DECISION VARIABLE

Looking at Fig. 1, we see that the filtered optical output of the Fabry-Perot filter  $s(t)$  is passed to a photodetector. The output current of the photodetector is  $\mathcal{R}|s(t)|^2$  where  $\mathcal{R}$  is the responsivity of the photodetector (A/W). Thus, we can see that the deterministic signal component of the decision variable at the integrator output is

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \quad (14)$$

We have already derived  $s_B(t)$ ,  $s_{ISI}(t)$ , and  $s_{ACI}(t)$  for  $0 \leq t \leq T$ . Directly substituting these three signals into Eq. (10) yields  $s(t)$  for  $0 \leq t \leq T$ . Now our true task is to compute  $|s(t)|^2$  and  $\int_0^T |s(t)|^2 dt$  as  $\mathcal{R}$  is an arbitrary constant.

$$s(t) = \underbrace{\sqrt{P}(1-\rho)}_K \underbrace{[s_B(t) + s_{ISI}(t) + s_{ACI}(t)]}_{s'(t)} \quad (15)$$

Then

$$|s(t)|^2 = K^2 s'(t) s'(t)^* \quad (16)$$

and dropping the  $(t)$  notation for convenience in the three terms of  $s'(t)$  and noting that complex conjugation is a linear operator

$$s'(t)^* = s_B^* + s_{ISI}^* + s_{ACI}^* \quad (17)$$

So

$$s'(t) s'(t)^* = (s_B + s_{ISI} + s_{ACI})(s_B^* + s_{ISI}^* + s_{ACI}^*) \quad (18)$$

Multiplication yields

$$\begin{aligned} s'(t) s'(t)^* &= s_B s_B^* + s_B s_{ISI}^* + s_B s_{ACI}^* \\ &\quad + s_{ISI} s_B^* + s_{ISI} s_{ISI}^* + s_{ISI} s_{ACI}^* \\ &\quad + s_{ACI} s_B^* + s_{ACI} s_{ISI}^* + s_{ACI} s_{ACI}^* \end{aligned} \quad (19)$$

Rearranging and simplifying yields

$$\begin{aligned}
|s(t)|^2 = K^2 s'(t) s'(t)^* &= K^2 \times \left[ \underbrace{s_B s_B^*}_{|s_B|^2} + \underbrace{s_{ISI} s_{ISI}^*}_{|s_{ISI}|^2} + \underbrace{s_{ACI} s_{ACI}^*}_{|s_{ACI}|^2} \right. \\
&\quad \left. + \underbrace{s_B s_{ISI}^* + s_{ISI} s_B^*}_{2\text{Re}[s_B s_{ISI}^*]} \right. \\
&\quad \left. + \underbrace{s_B s_{ACI}^* + s_{ACI} s_B^*}_{2\text{Re}[s_B s_{ACI}^*]} \right. \\
&\quad \left. + \underbrace{s_{ISI} s_{ACI}^* + s_{ACI} s_{ISI}^*}_{2\text{Re}[s_{ISI} s_{ACI}^*]} \right] \quad (20)
\end{aligned}$$

We will integrate each of these terms over 0 to  $T$  to compute  $\int_0^T |s(t)|^2 dt$ .

Substituting the appropriate definitions into the six terms of Eq. (20) yields 21 terms (“clusters of summations”) which are integrated from 0 to  $T$ . Also note that we had to create several “gating” functions to turn off integrals when the pulse function products in some of the 21 terms fail to overlap for certain summation indices. Although intuitively obvious, see Appendix A [below Eqs. (104) and (112)] for definitions of the max and min functions which appear in some of the forthcoming equations.

Recall that  $K = \sqrt{P}(1 - \rho)$  and  $K^2 = P(1 - \rho)^2$ . Then we have

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt \quad (21)$$

which has the following terms [see Eq. (20)]

$$K^2 \int_0^T |s_B|^2 dt = P(1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left( T - \left( \frac{\max(g, m)}{\beta} \right) \right) \quad (22)$$

$$K^2 \int_0^T |s_{ISI}|^2 dt = A_{ISI} + B_{ISI} + C_{ISI} \quad (23)$$

where

$$A_{ISI} = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0$$

in which

$$\varphi_0 = \rho^{g+m} b_{0,i} b_{0,r} \left( \frac{\min(g + (1+i)q, m + (1+r)q)}{\beta} \right)$$

$$B_{ISI} = 2P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g, i, m, r)$$

in which

$$G_1(g, i, m, r) = \begin{cases} \frac{1}{\beta} [(g + (1+i)q) - (m + rq)], & \text{for } m + rq < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

and

$$C_{ISI} = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \left[ T - \left( \frac{\max(g + iq, m + rq)}{\beta} \right) \right]$$

$$K^2 \int_0^T |s_{ACI}|^2 dt = A_{ACI} + B_{ACI} + C_{ACI} + D_{ACI} + E_{ACI} + F_{ACI} \quad (24)$$

where

$$A_{ACI} = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} \times \varphi_1$$

in which

$$\varphi_1 = \begin{cases} b_{k,0} b_{n,0}^* \frac{1}{j(\omega_k - \omega_n)} \times \dots \\ \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g, m) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right], & \text{for } k \neq n \\ \frac{\underbrace{b_{k,0} b_{k,0}^*}_{|b_{k,0}|^2}}{|b_{k,0}|^2} e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g, m) \right], & \text{for } k = n \end{cases}$$

$$B_{ACI} = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} \times \varphi_2$$

in which

$$\varphi_2 = \begin{cases} b_{k,\ell} b_{n,r}^* \left[ \frac{e^{j[-\omega_k(g/\beta) + \omega_n(m/\beta)]}}{j(\omega_k - \omega_n)} \left( e^{j[(\omega_k - \omega_n)(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)]} - 1 \right) \right], & \text{for } k \neq n \\ b_{k,\ell} b_{k,r}^* \left( e^{j[(\omega_k/\beta)(m-g)]} \left[ \frac{1}{\beta} \min(g + (1+\ell)q, m + (1+r)q) \right] \right), & \text{for } k = n \end{cases}$$

$$C_{ACI} = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} \times \varphi_3$$

in which

$$\varphi_3 = \begin{cases} b_{k,\ell} b_{n,r}^* \left[ \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g + \ell q, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], & \text{for } k \neq n \\ b_{k,\ell} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g + \ell q, m + rq) \right], & \text{for } k = n \end{cases}$$

$$D_{ACI} = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_4$$

in which

$$\varphi_4 = \left\{ \begin{array}{ll} \rho^{g+m} b_{k,0} b_{n,r}^* \left[ \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right] \\ + \rho^{g+m} b_{k,0}^* b_{n,r} \left[ \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{-j[(\omega_k - \omega_n)(1/\beta) \max(g, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], & \text{for } k \neq n \\ [b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}] \rho^{g+m} \times \dots \\ \left[ T - \frac{1}{\beta} \max(g, m + rq) \right], & \text{for } k = n \end{array} \right.$$

$$E_{ACI} = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_5$$

in which

$$\varphi_5 = \left\{ \begin{array}{l} \left[ \left( \rho^{g+m} b_{k,0} b_{n,r}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)((m+(1+r)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \right. \\ \left. \left. \left. - e^{j[(\omega_n/\beta)(m-g)]} \right] \right) \right. \\ \left. + \left( \rho^{g+m} b_{k,0}^* b_{n,r} \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)((m+(1+r)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \right. \\ \left. \left. \left. - e^{-j[(\omega_n/\beta)(m-g)]} \right] \right) \right] G_2(g, m, r), \quad \text{for } k \neq n \\ \\ [b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}] \rho^{g+m} \times \dots \\ \\ \left( \frac{1}{\beta} [(m + (1+r)q) - g] \right) G_2(g, m, r), \quad \text{for } k = n \end{array} \right.$$

where

$$G_2(g, m, r) = \left\{ \begin{array}{ll} 1, & \text{for } g < m + (1+r)q \\ 0, & \text{otherwise} \end{array} \right\}$$

$$F_{ACI} = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_6$$

in which

$$\varphi_6 = \left\{ \begin{array}{ll} \left( \rho^{g+m} b_{k,\ell} b_{n,r}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)((g+(1+\ell)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)((m+rq)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right. \\ \left. + \rho^{g+m} b_{k,\ell}^* b_{n,r} \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)((g+(1+\ell)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{-j[(\omega_k - \omega_n)((m+rq)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right) G_3(g, \ell, m, r), & \text{for } k \neq n \\ \left[ b_{k,\ell} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,\ell}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \dots \\ \left( \frac{1}{\beta} [(g + (1 + \ell)q) - (m + rq)] \right) G_3(g, \ell, m, r), & \text{for } k = n \end{array} \right\}$$

where

$$G_3(g, \ell, m, r) = \begin{cases} 1, & m + rq < g + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} K^2 \int_0^T 2 \operatorname{Re}[s_B s_{ISI}^*] dt &= K^2 \int_0^T 2[s_B s_{ISI}] dt = \\ &2P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} G_4(g, i, m) \\ &+ 2P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} \left[ T - \frac{1}{\beta} \max(g, m + iq) \right] \end{aligned} \quad (25)$$

where

$$G_4(g, i, m) = \begin{cases} \frac{1}{\beta} [(m + (1 + i)q) - g], & g < m + (1 + i)q \\ 0, & \text{otherwise} \end{cases}$$



$$K^2 \int_0^T 2\text{Re}[s_B^* s_{ACI}] dt = K^2 \int_0^T 2\text{Re}[s_B s_{ACI}] dt = A_{B-ACI} + B_{B-ACI} + C_{B-ACI} \quad (26)$$

where

$$A_{B-ACI} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_7$$

in which

$$\begin{aligned} \varphi_7 = & \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m)-m)/\beta)} \right] \\ & + \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m)-m)/\beta)} \right] \end{aligned}$$

$$B_{B-ACI} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_8$$

in which

$$\begin{aligned} \varphi_8 = & \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k((1+\ell)q/\beta)} - e^{j\omega_k((g-m)/\beta)} \right] G_5(g, \ell, m) \\ & + \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k((1+\ell)q/\beta)} - e^{-j\omega_k((g-m)/\beta)} \right] G_5(g, \ell, m) \end{aligned}$$

where

$$G_5(g, \ell, m) = \begin{cases} 1, & g < m + (1+\ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$C_{B-ACI} = P(1-\rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_9$$

in which

$$\begin{aligned}\varphi_9 &= \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right] \\ &+ \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right]\end{aligned}$$

$$\begin{aligned}K^2 \int_0^T 2\text{Re}[s_{ISI}^* s_{ACI}] dt &= K^2 \int_0^T 2\text{Re}[s_{ISI} s_{ACI}] dt \\ &= A_{ISI-ACI} + B_{ISI-ACI} + C_{ISI-ACI} \\ &+ D_{ISI-ACI} + E_{ISI-ACI} + F_{ISI-ACI}\end{aligned}\quad (27)$$

where

$$A_{ISI-ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{10}$$

in which

$$\begin{aligned}\varphi_{10} &= \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - 1 \right] G_6(g, i, m) \\ &+ \rho^{g+m} b_{0,i} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - 1 \right] G_6(g, i, m)\end{aligned}$$

where

$$G_6(g, i, m) = \begin{cases} 1, & m < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

$$B_{ISI-ACI} = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{11}$$

in which

$$\begin{aligned}\varphi_{11} &= \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta) \min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{j\omega_k[-m/\beta]} \right] \\ &+ \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta) \min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{-j\omega_k[-m/\beta]} \right]\end{aligned}$$

$$C_{ISI-ACI} = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{12}$$

in which

$$\begin{aligned} \varphi_{12} = & \left[ \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{j\omega_k[\ell q/\beta]} \right] \right. \\ & \left. + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{-j\omega_k[\ell q/\beta]} \right] \right] G_7(g, i, \ell, m) \end{aligned}$$

where

$$G_7(g, i, \ell, m) = \begin{cases} 1, & m + \ell q < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

$$D_{ISI-ACI} = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{13}$$

in which

$$\begin{aligned} \varphi_{13} = & \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta) \max(g+iq, m)-m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta) \max(g+iq, m)-m/\beta]} \right] \end{aligned}$$

$$E_{ISI-ACI} = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{14}$$

in which

$$\begin{aligned} \varphi_{14} = & \left[ \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(1+\ell)q]} - e^{j\omega_k[(1/\beta)(g+iq)-m/\beta]} \right] \right. \\ & \left. + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(1+\ell)q]} - e^{-j\omega_k[(1/\beta)(g+iq)-m/\beta]} \right] \right] G_8(g, i, \ell, m) \end{aligned}$$

where

$$G_8(g, i, \ell, m) = \begin{cases} 1, & g + iq < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$F_{ISI-ACI} = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{15}$$

in which

$$\begin{aligned} \varphi_{15} = & \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k [T - (m/\beta)]} - e^{j\omega_k [(1/\beta) \max(g+iq, m+\ell q) - m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k [T - (m/\beta)]} - e^{-j\omega_k [(1/\beta) \max(g+iq, m+\ell q) - m/\beta]} \right] \end{aligned} \quad (28)$$

We have now laid down all terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt$$

This will be used to compute the deterministic signal detection statistic  $X$  where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

This will be used to compute probabilities of bit error for the dense WDM system.

#### D. THE DECISION VARIABLE $Y$

The decision variable  $Y$  appears at the output of the integrator (see Fig. 1). It consists of a signal component  $X$ , which was presented in Section C, and a noise component  $N$

$$Y = X + N \quad (29)$$

where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \quad (30)$$

and

$$N = \int_0^T n(t)dt \quad (31)$$

We note that  $n(t)$  is a zero mean Gaussian random process with two-sided current spectral density  $N_0$  ( $A^2/Hz$ ), and  $N$  is a zero mean Gaussian random variable with variance  $N_0T$ .  $Y$  is compared to a threshold  $V_T$  to determine whether a "0" or "1" bit is presented at the output.

### E. PROBABILITY OF BIT ERROR FOR THE DENSE WDM SYSTEM

For a detection threshold  $V_T$  and an ACI/ISI bit pattern  $\psi_p = \{b_{k,\ell}, b_{0,i}\}$  for  $k = -M/2, \dots, +M/2, k \neq 0, \ell = -L, \dots, 0$ , and  $i = -L_0, \dots, -1$ ; the conditional probability of bit error for the dense WDM system utilizing the decision variable  $Y$  described by Eqs. (28)–(30) is given by [2, 4]

$$P_e(\psi_p) = \frac{1}{2}Q\left(\frac{X_1(\psi_p) - V_T}{\sqrt{N_0T}}\right) + \frac{1}{2}Q\left(\frac{V_T - X_0(\psi_p)}{\sqrt{N_0T}}\right) \quad (32)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \quad (33)$$

$X_0$  is the value of  $X$  given by Eq. (14) or (30) with the value of  $\int_0^T |s(t)|^2 dt$  obtained by summing Eqs. (22)–(27) when  $b_{0,0} = 0$ .  $X_1$  is obtained in the same manner with  $b_{0,0} = 1$ . The average probability of bit error  $P_b$  is given by taking the expected value of  $P_e(\psi_p)$  given in Eq. (32) over all possible bit patterns  $\psi_p$  [2]. Let us define an ACI/ISI bit pattern set  $\psi = \{\psi_p\}$ ;  $p = 1, \dots, NPAT$ .  $NPAT$  is the total possible number of bit patterns in the set  $\psi = \{\psi_p\}$ . Then

$$P_b = E_{\{\psi_p\}} \{P_e(\psi_p)\} \quad (34)$$

If we count up the number of independent bits in  $\psi_p$ , there are  $M(L+1) + L_0$  bits. Using the assumption that a 0 or 1 is equally probable, we see that there are two

possible ways to fill each bit position. Then, by the *generalized multiplication rule*, if we let  $NPAT$  be the total number of possible bit patterns in the set  $\psi = \{\psi_p\}$

$$NPAT = (2)_1(2)_2 \cdots (2)_{M(L+1)+L_0} = 2^{M(L+1)+L_0} \quad (35)$$

If we assume that all bit patterns in  $\psi = \{\psi_p\}$  are equiprobable (an excellent assumption), then by the law of total probability

$$P_b = \frac{1}{2^{M(L+1)+L_0}} P_e(\psi_1) + \frac{1}{2^{M(L+1)+L_0}} P_e(\psi_2) + \cdots + \frac{1}{2^{M(L+1)+L_0}} P_e(\psi_{NPAT})$$

and

$$P_b = \frac{1}{2^{M(L+1)+L_0}} \sum_{p=1}^{NPAT} P_e(\psi_p) \quad (36)$$

### III. NUMERICAL RESULTS

We attempt to generate four probability of bit error graphs. We limit our expression for  $X$  given by Eq. (14) or (29) and the sum of Eqs. (22)–(27) by setting

$$L_0 = 1 \quad L = 0 \quad \phi_k = 0$$

and

$$\omega_k = \frac{2\pi k I}{T}$$

where  $k$  is an integer and  $I$  is the normalized channel spacing integer ( $I > 0$ ). When we do this, our 21 terms (“clusters of summations”) reduce to ten terms (“clusters of summations”). In the interests of brevity, we refer the reader to Appendix B for the details on how we came up with the ten programmable terms of  $X$  for this case. Referring to Appendix B, the probability of bit error equations we use to generate these graphs mirror the equations presented in Chapter II, Section E. Referring to Appendix C, one finds the programming strategy for the ten terms and the final graphs, the actual programs for each of the ten terms, and the final four graphing programs which produce one graph each. Each of the four graphs plot probability of bit error versus signal-to-noise ratio  $Z = \mathcal{R} P \sqrt{T/N_0}$  in dB. Each graph has five traces. One trace shows either Single Channel (SC) operation without Fabry-Perot (FP) filtering or SC operation with FP filtering and without ISI and ACI for comparison with the other four dense WDM traces. The other four traces show probability of bit error versus  $Z$  (dB) for four selected values of the normalized channel spacing integer  $I$ , or equivalently the number of adjacent channels  $M$ . See Appendix C for the relationship between  $I$  and  $M$ . Each of the four graphs is plotted for a different value  $\beta T$ , the free-spectral range-bit period product. We also note that the four

graphs are generated with

$$\rho = \text{Single Cavity Fabry - Perot Filter Power Reflectivity} = 0.99$$

We now present the values of  $\beta T$  the four values of  $M$ , and the corresponding values of  $I$  used for each of the four graphs. Each separate value of  $M$ , or equivalently  $I$ , produces one of the four dense WDM traces for use in comparison with the fifth trace showing single channel operation.

For Fig. 2:  $\beta T = 500$  and

$$M = \begin{bmatrix} \underbrace{124}_{I=4} & \underbrace{98}_{I=5} & \underbrace{60}_{I=8} & \underbrace{24}_{I=20} \end{bmatrix}$$

For Fig. 3:  $\beta T = 1000$  and

$$M = \begin{bmatrix} \underbrace{198}_{I=5} & \underbrace{164}_{I=6} & \underbrace{110}_{I=9} & \underbrace{48}_{I=20} \end{bmatrix}$$

For Fig. 4:  $\beta T = 1500$  and

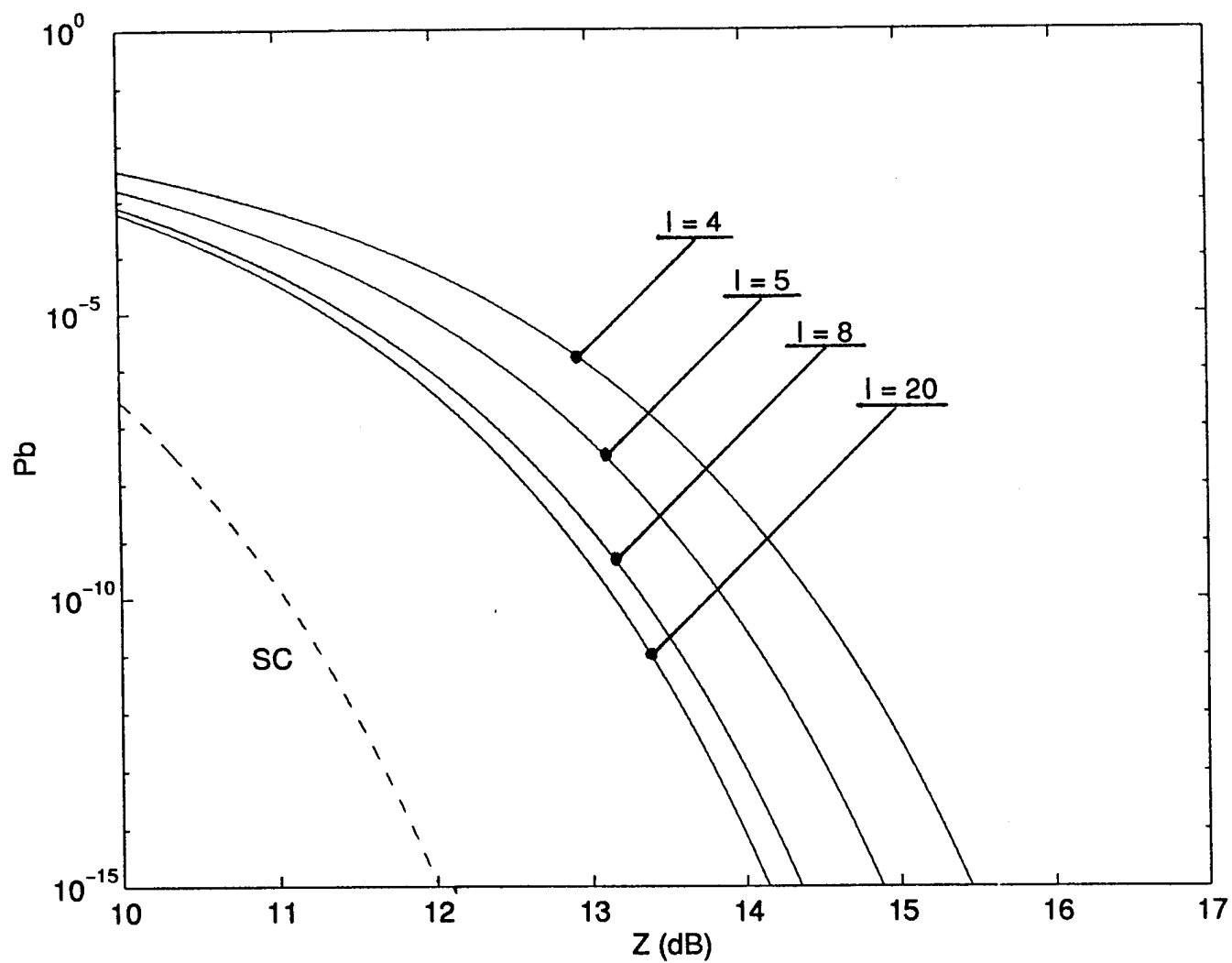
$$M = \begin{bmatrix} \underbrace{212}_{I=7} & \underbrace{164}_{I=9} & \underbrace{124}_{I=12} & \underbrace{74}_{I=20} \end{bmatrix}$$

For Fig. 5:  $\beta T = 2000$  and

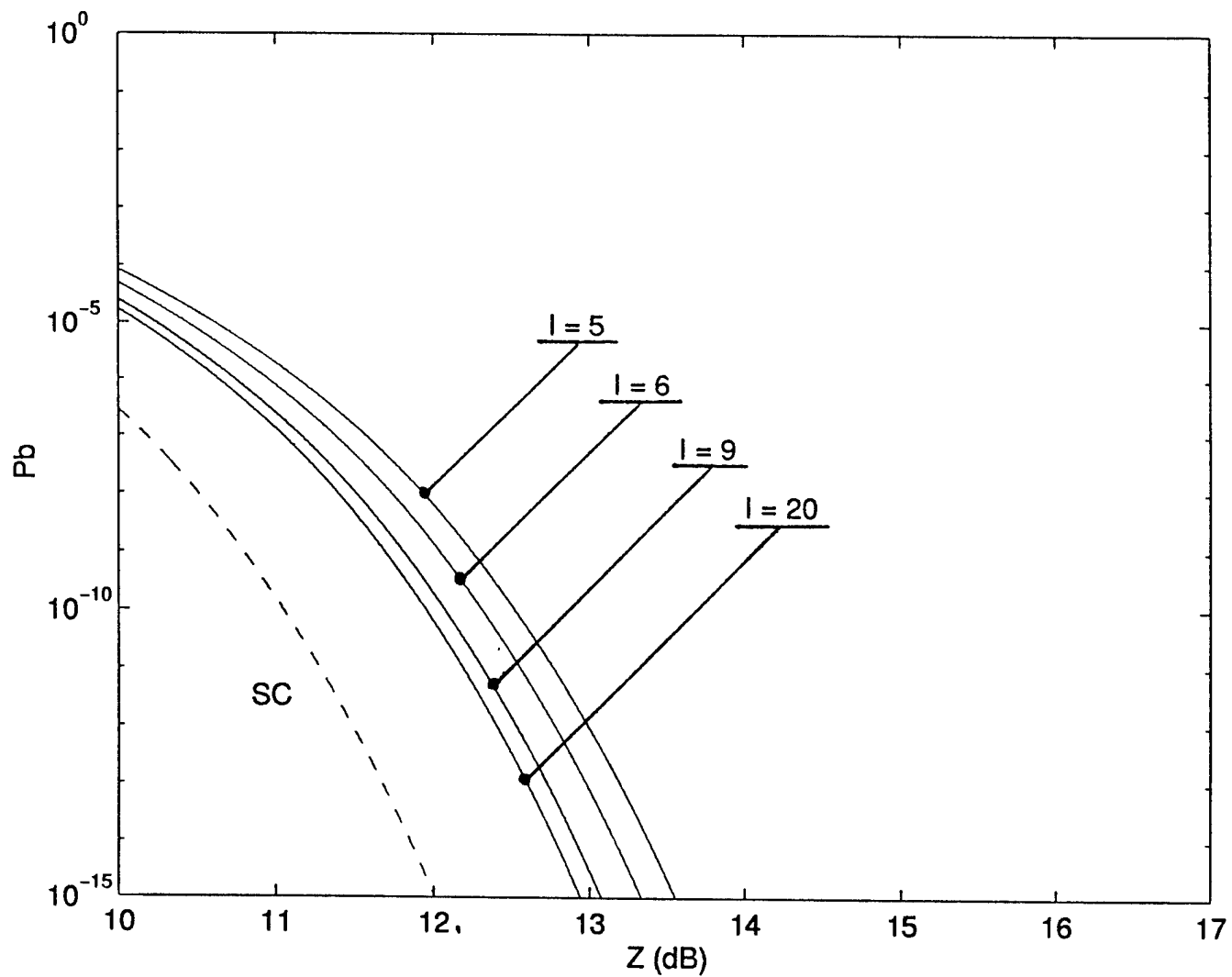
$$M = \begin{bmatrix} \underbrace{248}_{I=8} & \underbrace{220}_{I=9} & \underbrace{164}_{I=12} & \underbrace{98}_{I=20} \end{bmatrix}$$

Looking at Figs. 2-5, we see that only Fig. 2 ( $\beta T = 500$ ) is a complete graph. For  $\beta T = 500$  we were able to compute all ten terms of  $X$  (see Appendix B) for all four given values of  $M$ , or equivalently  $I$ , and completely compute and simulate all ISI and ACI effects of our complete model given the constraints given at the beginning of this chapter ( $L_0 = 1$ ,  $L = 0$ ,  $\phi_k = 0$ ,  $\omega_k = 2\pi kI/T$ ). For Figs. 3-5 ( $\beta T = 1000$ , 1500, and 2000, respectively), we were unable to arrive at a solution for Eq. (24) (see Term #5 in Appendix B), which is the very important  $K^2 \int_0^T |s_{ACI}|^2 dt$  term. It is very computationally intensive as it involves computing a quadruple summation multiple times

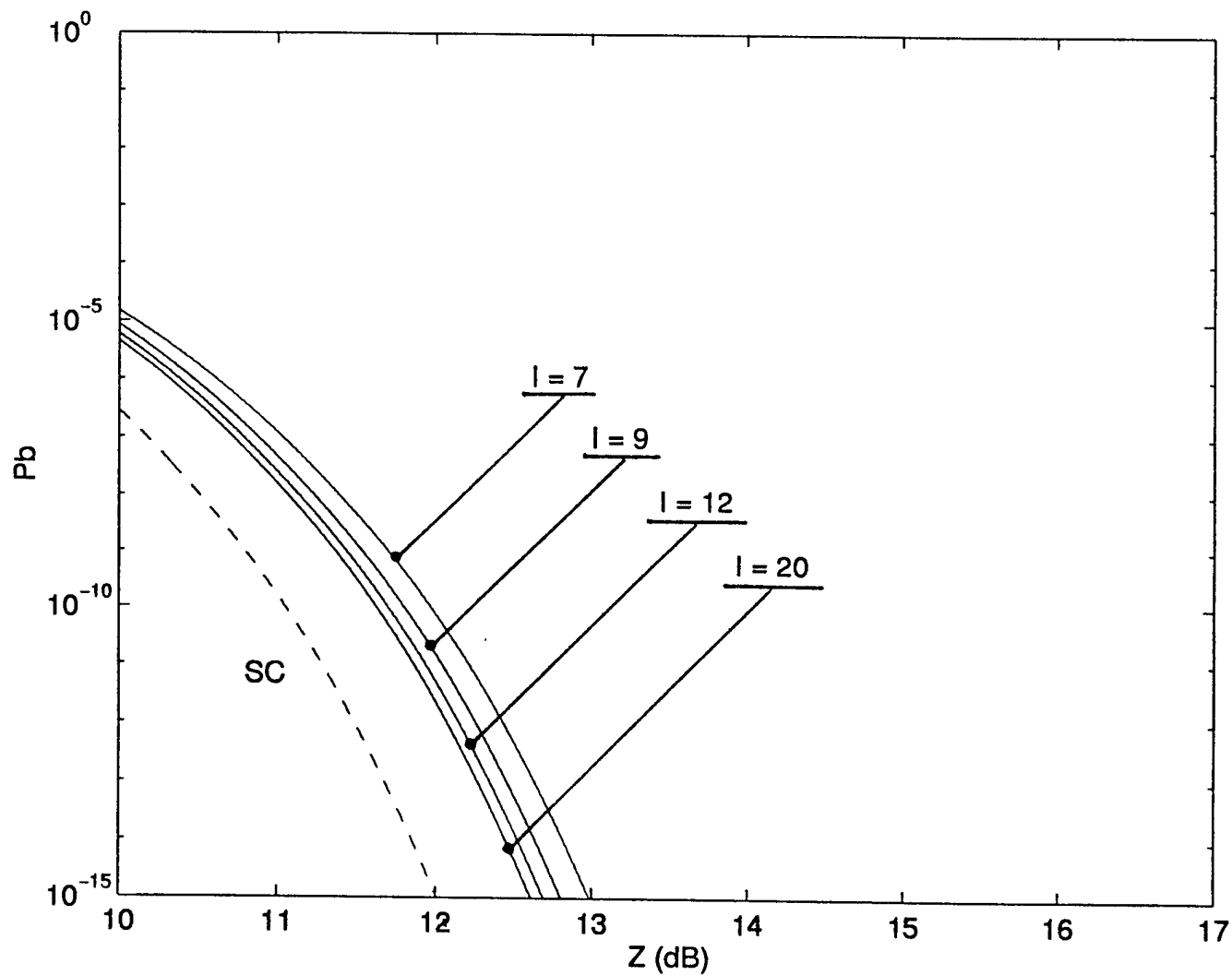




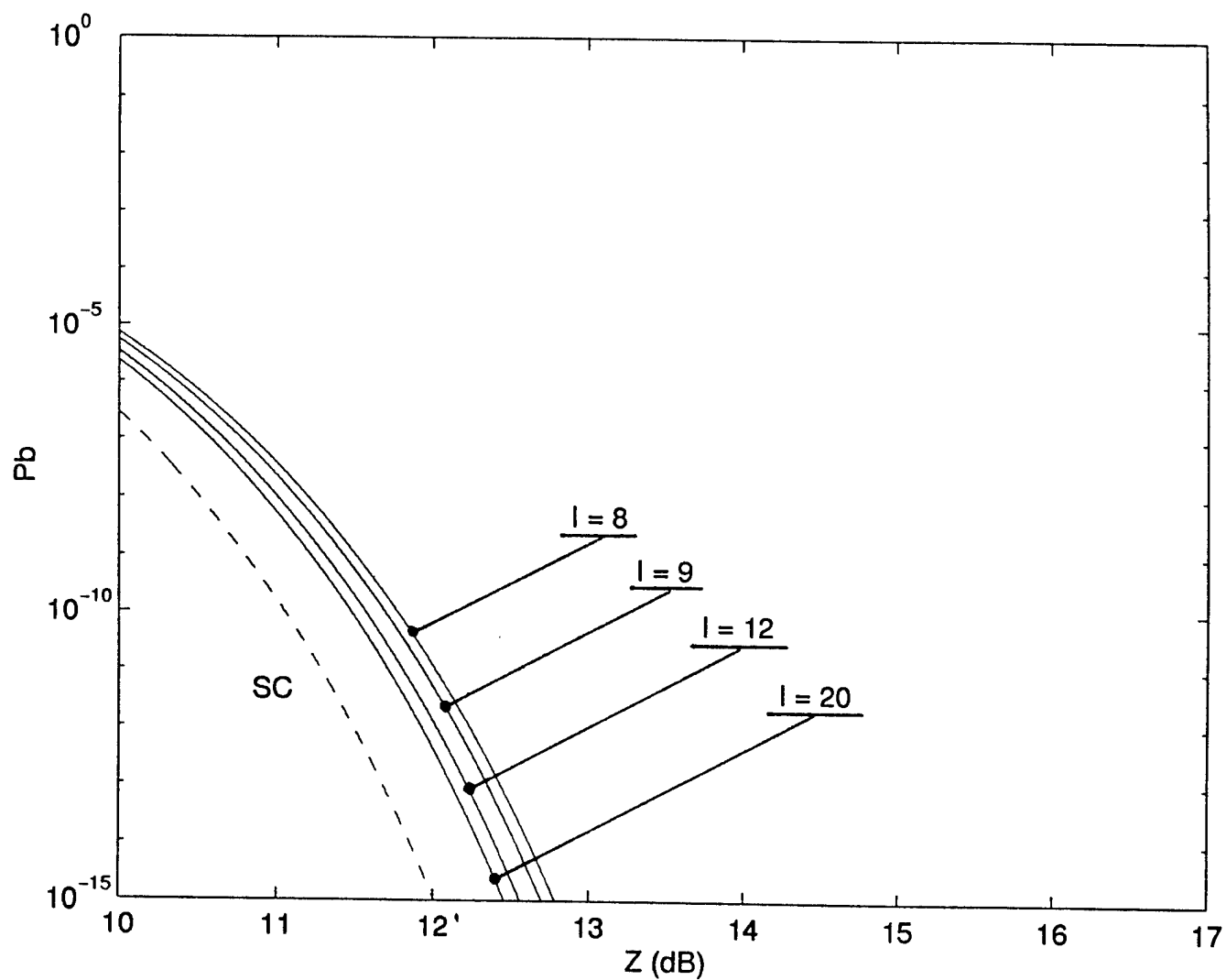
**Figure 2:** Probability of bit error vs.  $Z$  (db) for  $\beta T = 500$ .



**Figure 3:** Probability of bit error vs.  $Z$  (db) for  $\beta T = 1000$ . Only  $I = 20$  trace accurate (within 1/16 dB).



**Figure 4:** Probability of bit error vs  $Z$  (db) for  $\beta T = 1500$ . Only  $I = 20$  trace accurate (within 1/16 dB).



**Figure 5:** Probability of bit error vs.  $Z$  (dB) for  $\beta T = 2000$ . Only  $I = 20$  trace accurate (within 1/5 dB).

for the various adjacent channel bit “loadings” (see Appendix B for the details), as well as having to repeat the same very lengthy calculations for each of the four given values of  $M$ , or equivalently  $I$ . However, we will still be able to draw some conclusions for the  $I = 20$  traces of these graphs as the channel frequency spacing between adjacent channels given by

$$\Delta f = \frac{\omega_k}{2\pi k} = \frac{I}{T} = IR_b \quad (37)$$

where  $R_b$  is the channel data rate or channel bit rate. Note that  $\Delta f$  is proportional to  $I$ . At these larger frequency spacings, ACI effects become quite negligible. Equation (37) explains why we call  $I$  the normalized channel spacing integer as adjacent channels are separated from each other at an integer multiple  $I$  of the bit rate  $R_b$ . Let us assume a system design of a filter free-spectral-range  $\beta = 3800 \times 10^9$  Hz = 3.8 GHz. Then we can say that for  $\beta T = 500$  our bit rate  $R_b = 7.6$  Gb/sec, for  $\beta T = 1000$  our bit rate is  $R_b = 3.8$  Gb/sec, for  $\beta T = 1500$  our bit rate is  $R_b \simeq 2.5$  Gb/sec, and for  $\beta T = 2000$  our bit rate is  $R_b = 1.9$  Gb/sec. It is noteworthy that, in the time domain model, calculating probability of bit error as presented in Chapter II and Appendix A is easiest for the highest channel data rate  $R_b$ , or equivalently the smallest bit period  $T$ .

We define power penalty as the increase in signal-to-noise ratio required for the dense WDM system to achieve a  $10^{-15}$  bit error probability over the Single Channel (SC) system which achieves this goal operating at  $Z = 12$  dB. We can now make some power penalty statements.

Looking at Fig. 2 ( $\beta T = 500$ ) we see that the power penalty increases as  $I$  decreases, or equivalently the number of adjacent channels  $M$  increases. Looking at Eq. (37), we see that as  $I$  decreases, the interchannel frequency spacing  $\Delta f$  decreases. Thus, it is logical that ACI effects would increase. In short, the channels become much

more tightly “packed”. Looking at the results for Term #5 in Appendix C when  $\beta T = 500$  we see that the magnitude of the contribution for  $I = 20$  is only about two to four percent of the contribution for  $I = 4$ . Returning to Fig. 2 ( $\beta T = 500$ ), we see that the power penalty for  $I = 20$  and  $M = 24$  is approximately  $2\frac{1}{8}$  dB; for  $I = 8$  and  $M = 60$ , the penalty is approximately  $2\frac{3}{8}$  dB; for  $I = 5$  and  $M = 98$ , the penalty is approximately  $2\frac{7}{8}$  dB; and finally for  $I = 4$  and  $M = 124$  the penalty is approximately  $3\frac{1}{2}$  dB. It is in this case of  $I = 4$  and  $M = 124$  that we see the true beauty of dense WDM. If we assume  $\beta = 3.8$  GHz, then the channel bit rate  $R_b$  is 7.6 Gb/sec, and the aggregate system bit rate  $R_b$  is  $124 \times 7.6$  Gb/s = 942 Gb/sec, which is quite close to 1 Tb/sec ( $1 \text{ Tb/sec} = 10^{12} \text{ b/sec}$ ). Thus, for slightly more than doubling the signal-to-noise ratio  $Z = \mathcal{R} P \sqrt{T/N_0}$ , we have increased the aggregate bit rate of one single fiber close to a terabit per second. Also, the dense WDM system utilizes a noncoherent receiver and, thus, is much cheaper to implement than a coherent optical FDMA network with expensive synchronization circuitry. Looking at  $Z$ , however, we see that increasing signal-to-noise ratio is not that easy to do. Upper limits have already been approached in making low-noise current amplifiers; thus, the true technical problem to be solved in the dense WDM system utilizing single-cavity Fabry-Perot filters is to place an optical amplifier before the FP filter (see Fig. 1) to boost the received optical power  $P$ . With this problem solved, economical dense WDM systems utilizing single-cavity FP filters with aggregate bit rates with many terabits per second are possible.

Now, in Figs. 3–5 we have already seen that at  $I = 20$ , the effects of ACI become quite negligible as the channels are spaced farther apart in frequency. Thus, we can conclude that the  $I = 20$  probability of bit error traces are accurate for  $\beta T = 1000, 1500$ , and  $2000$ . The rest of the traces for  $I < 20$  are not. A manuscript in preparation by Tri T. Ha entitled, “A Discrete Time Approach to Sensitivity Analysis

of Direct Detection Optical FDMA Networks with OOK Modulation,” considers the same problem as this thesis, but uses a discrete time approach. The probability of bit error graphs in this future paper are generated with the same parameters used for this thesis. For the discrete time approach, the  $N = \beta T = 1000$  and 1500 traces for  $I = 20$  match the  $I = 20$  traces for this thesis to within less than  $1/16$  of a dB, and the  $N = \beta T = 2000$  trace for  $I = 20$  matches the one in this thesis to within  $1/5$  of a dB.

Finally, using the case  $\beta T = 500$  we feel that our time domain model is very accurate as Fig. 2 agrees with the corresponding discrete time graph for  $N = 500$  to within  $1/4$  of a dB for  $I = 4$  and  $M = 124$ , agrees to within  $1/8$  of a dB for  $I = 5$  and  $M = 98$ , and matches almost exactly for  $I = 8$  and  $M = 60$ , and  $I = 20$  and  $M = 24$ . These differences are well within the realistic bounds of numerical error as our time domain approach requires many billions of calculations.





## IV. CONCLUSION

We have presented a complete model and equation for the probability of bit error of an OOK, dense WDM system employing single-cavity Fabry-Perot filters as channel demultiplexers. Our expression completely models Inter-Symbol Interference (ISI), Adjacent Channel Interference (ACI), and phase offset between Channel 0 and Channel  $k$  for all  $M$  of the adjacent channels. No simplifying mathematical assumptions have been made to arrive at the final answer contained in Eq. (14), Eq. (29), and the sum of Eqs. (22)–(27). Others [1, 3] have worked on this problem; however, the solution is arrived at in the frequency domain and often involves simplifying mathematical assumptions [1, Eqs. (15), (36), (39) and 3, Eq. (5)]. The closed form expression we have presented for probability of bit error for the dense WDM system, although mathematically rigorous and an extremely complete model, has one significant drawback—it is extremely computationally intensive. The single complete graph for probability of bit error when  $\beta T = 500$  took three months to compute using multiple *SPARC 10* workstations working 24 hours per day. Even for our limited case of  $L_0 = 1$ ,  $L = 0$ ,  $\omega_k = 2\pi kI/T$ , and  $\phi_k = 0$ , we never arrived at solutions for the other three values of  $\beta T = 1000, 1500$ , and  $2000$ . The solution is to use a discrete time approach. Again, we are confident of the accuracy of our model as the same problem was attacked independently via discrete time approach, and our graph for  $\beta T = 500$  matched the discrete case of  $N = \beta T = 500$  to within the bounds of numerical error. Finally, we again mention that to practically implement dense WDM systems with aggregate bit rates of many terabits per second, large increases (doubling, tripling, ...) in the SNR of the dense WDM system over that required for a

single channel system are necessary. To accomplish this an optical amplifier will have to be placed in front of the Fabry-Perot filter to greatly increase the filter's received optical power.

## APPENDIX A

### DERIVATION OF $\int_0^T |s(t)|^2 dt$ FOR USE IN $X = \mathcal{R} \int_0^T |s(t)|^2 dt$

Before we begin the derivation of  $\int_0^T |s(t)|^2 dt$ , we must first define the variables, equations, and terms involved:

$Y$  — The decision variable appearing at the output of the integrator (see Fig. 1) according to

$$Y = X + N$$

$X$  — The deterministic signal portion of the decision variable

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

$\mathcal{R}|s(t)|^2$  — Output current of the photodetector

$\mathcal{R}$  — Responsivity of the photodetector (A/W)

$n(t)$  — Postdetection thermal noise with two-sided current spectral density  $N_0$  (A<sup>2</sup>/Hz)

$N$  — Random signal component of the decision variable  $Y$ .  $N$  is a zero-mean, Gaussian random variable with variance  $N_0 T$

$$N = \int_0^T n(t) dt$$

$\rho$  — Power reflectivity of the single-cavity Fabry-Perot filter

$\beta$  — Free spectral range of the single-cavity Fabry-Perot filter (Hz)

$T$  — Data bit period (s)

$P$  — Received optical power (W)

$$p_T(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{otherwise} \end{cases} \quad (38)$$

Desired Channel/Channel of Interest — Channel 0

Adjacent Channel — Channel  $k$ ,  $k = -M/2, \dots, -1, 1, \dots, +M/2$  where  $M$  is an even integer

$b_{0,0}$  — The detected bit of interest in the interval  $0 \leq t \leq T$

$b_{0,i}$  — Bit in Channel 0 during the  $i^{\text{th}}$  time interval  $[iT, (i+1)T]$  where

$$b_{0,i} \in \{0, 1\}$$

$b_{k,\ell}$  — Bit in Channel  $k$  during the  $\ell^{\text{th}}$  time interval  $[\ell T, (\ell+1)T]$  where

$$b_{k,\ell} \in \{0, e^{j\phi_k}\}$$

and

$$j = \sqrt{-1}$$

$\phi_k$  — Phase offset between Channel 0 and Channel  $k$ .  $\phi_k$  is assumed to be a uniformly distributed random variable between  $[0, 2\pi]$

$$\phi_k \sim U[0, 2\pi]$$

$\omega_k$  — Radian frequency spacing between Channel 0 and Channel  $k$ . Channels are symmetric around Channel 0, i.e.,

$$\omega_k = -\omega_{-k}$$

$h(t)$  — Impulse response of the Channel 0 single cavity Fabry-Perot filter

$$h(t) = (1 - \rho) \sum_{g=0}^{\infty} \rho^g \delta\left(t - \frac{g}{\beta}\right) \quad (39)$$

$\delta(t)$  — The *Dirac Delta Function* defined in two parts

$$\delta(t) = \left\{ \begin{array}{ll} 0, & t < 0 \\ 0, & t > 0 \end{array} \right\} \quad (40a)$$

and

$$\int_{0-}^{0+} \delta(t) dt = 1 \quad (40b)$$

$b_0(t)$  — Complex baseband equivalent data signal in Channel 0

$$b_0(t) = \sum_{i=-L_0}^0 b_{0,i} p_T(t - iT) \quad (41)$$

$L_0$  — An integer greater than zero representing the number of bits in Channel 0 that are trailing the detected bit  $b_{0,0}$ .

$b_k(t)$  — Complex baseband equivalent data signal in Channel  $k$

$$b_k(t) = \sum_{\ell=-L}^0 b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \quad (42)$$

$L$  — An integer greater than zero representing the number of bits in Channel  $k$  which trail the 0<sup>th</sup> bit in Channel  $k$ ,  $b_{k,0}$ .

$r(t)$  — Received complex baseband signal at input of the Channel 0 Fabry-Perot filter

$$r(t) = \sqrt{P} b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sqrt{P} b_k(t) \quad (43)$$

We now begin the derivation of  $\int_0^T |s(t)|^2 dt$  which allows the computation of

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

where  $s(t)$  is the output of the Fabry-Perot filter with an impulse response of  $h(t)$

$$\xrightarrow{r(t)} \boxed{h(t)} \xrightarrow{s(t)}$$

$$h(t) = (1 - \rho) \sum_{g=0}^{\infty} \rho^g \delta \left( t - \frac{g}{\beta} \right)$$

As stated before

$$r(t) = \sqrt{P} b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sqrt{P} b_k(t)$$

Substituting Eqs. (41) and (42) yields

$$r(t) = \sqrt{P} \left( \sum_{i=-L_0}^0 b_{0,i} p_T(t - iT) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \right) \quad (44)$$

We are interested in the detected bit  $b_{0,0}$  during the detection interval  $0 \leq t \leq T$ , so we need only evaluate  $s(t) = r_0(t)$  in the interval  $0 \leq t \leq T$

$$s(t) = r_0(t) = r(t) * h(t) = \int_{-\infty}^{\infty} h(t - \tau) r(\tau) d\tau = \int_{-\infty}^{\infty} r(t - \tau) h(\tau) d\tau \quad (45)$$

$$\begin{aligned} s(t) = r_0(t) &= r(t) * h(t) \\ &= h(t) * r(t) \\ &= h(t) * \left( \sqrt{P} b_0(t) + \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} b_k(t) \right) \end{aligned} \quad (46)$$

As convolution is a linear operator and distributes over addition

$$s(t) = r_0(t) = \left( \sqrt{P} (h(t) * b_0(t)) + \sqrt{P} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} h(t) * b_k(t) \right) \quad (47)$$

$$s(t) = r_0(t) = \sqrt{P} \left( h(t) * b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} h(t) * b_k(t) \right) \quad (48)$$

Now looking at the term within the parentheses

$$\begin{aligned}
h(t) * b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} h(t) * b_k(t) &= h(t) * \sum_{i=-L_0}^0 b_{0,i} p_T(t - iT) \\
+ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} h(t) * \sum_{\ell=-L}^0 b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) & \quad (49)
\end{aligned}$$

Applying the distributive property of convolution

$$\begin{aligned}
h(t) * b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} h(t) * b_k(t) &= \sum_{i=-L_0}^0 h(t) * (b_{0,i} p_T(t - iT)) \\
+ \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 h(t) * \left( b_{k,\ell} e^{j\omega_k t} p_T(t - \ell T) \right) & \quad (50)
\end{aligned}$$

Expressing convolution in terms of its integral definition

$$\begin{aligned}
h(t) * b_0(t) + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} h(t) * b_k(t) &= \underbrace{\sum_{i=-L_0}^0 \int_{-\infty}^{\infty} h(t - \tau) [b_{0,i} p_T(\tau - iT)] d\tau}_{A_1} \\
+ \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \int_{-\infty}^{\infty} h(t - \tau) [b_{k,\ell} e^{j\omega_k \tau} p_T(\tau - \ell T)] d\tau}_{B_1} & \quad (51)
\end{aligned}$$

Substituting the expression for  $h(t - \tau)$  [Eq. (39)] into term  $A_1$ , above

$$A_1 = \sum_{i=-L_0}^0 \int_{-\infty}^{\infty} (1 - \rho) \sum_{g=0}^{\infty} \rho^g \delta \left( t - \tau - \frac{g}{\beta} \right) b_{0,i} p_T(\tau - iT) d\tau \quad (52)$$

Factoring  $(1 - \rho)$  and interchanging the order of integration and summation yields

$$A_1 = (1 - \rho) \sum_{i=-L_0}^0 \sum_{g=0}^{\infty} \rho^g b_{0,i} \int_{-\infty}^{\infty} \delta \left( t - \tau - \frac{g}{\beta} \right) p_T(\tau - iT) d\tau \quad (53)$$

Using the identity:  $\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$ , the integral evaluates to  $p_T(t - g/\beta - iT)$ . We then have the following for  $A_1$

$$\begin{aligned} A_1 &= \sum_{i=-L_0}^0 \int_{-\infty}^{\infty} h(t-\tau) [b_{0,i} p_T(\tau - iT)] d\tau \\ &= (1-\rho) \sum_{i=-L_0}^0 \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left( t - \frac{g}{\beta} - iT \right) \end{aligned} \quad (54)$$

Recalling the expression for

$$B_1 = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \int_{-\infty}^{\infty} h(t-\tau) [b_{k,\ell} e^{j\omega_k \tau} p_T(\tau - \ell T)] d\tau \quad (55)$$

Substituting the expression for  $h(t-\tau)$  given by Eq. (39) into Eq. (55) above, we obtain

$$B_1 = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \int_{-\infty}^{\infty} (1-\rho) \sum_{g=0}^{\infty} \rho^g \delta \left( t - \tau - \frac{g}{\beta} \right) [b_{k,\ell} e^{j\omega_k \tau} p_T(\tau - \ell T)] d\tau \quad (56)$$

Factoring  $1-\rho$ , interchanging the order of integration and summation, and factoring  $b_{k,\ell}$  and  $\rho^g$  from the integral

$$B_1 = (1-\rho) \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} \int_{-\infty}^{\infty} \delta \left( t - \tau - \frac{g}{\beta} \right) e^{j\omega_k \tau} p_T(\tau - \ell T) d\tau \quad (57)$$

Applying the property  $\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$

$$\int_{-\infty}^{\infty} \delta \left( t - \tau - \frac{g}{\beta} \right) e^{j\omega_k \tau} p_T(\tau - \ell T) d\tau = e^{j\omega_k (t - (g/\beta))} p_T \left( t - \frac{g}{\beta} - \ell T \right) \quad (58)$$

Substituting Eq. (58) into Eq. (57) yields

$$\begin{aligned} B_1 &= \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \int_{-\infty}^{\infty} h(t-\tau) [b_{k,\ell} e^{j\omega_k \tau} p_T(\tau - \ell T)] d\tau \\ &= (1-\rho) \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k (t - (g/\beta))} p_T \left( t - \frac{g}{\beta} - \ell T \right) \end{aligned} \quad (59)$$



Substituting Eq. (54) and Eq. (59) into Eq. (51) and then substituting this result into Eq. (48) yields

$$\begin{aligned}
s(t) = r_0(t) = \sqrt{P} & \left[ \underbrace{(1-\rho) \sum_{i=-L_0}^0 \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left( t - \frac{g}{\beta} - iT \right)}_{A_1} \right. \\
& + \underbrace{(1-\rho) \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left( t - \frac{g}{\beta} - \ell T \right)}_{B_1} \left. \right] \quad (60)
\end{aligned}$$

We can now factor out  $(1-\rho)$  from our expression above. Then

$$\begin{aligned}
s(t) = r_0(t) = \sqrt{P} (1-\rho) & \left[ \sum_{i=-L_0}^0 \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left( t - \frac{g}{\beta} - iT \right) \right. \\
& + \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left( t - \frac{g}{\beta} - \ell T \right) \left. \right] \quad (61)
\end{aligned}$$

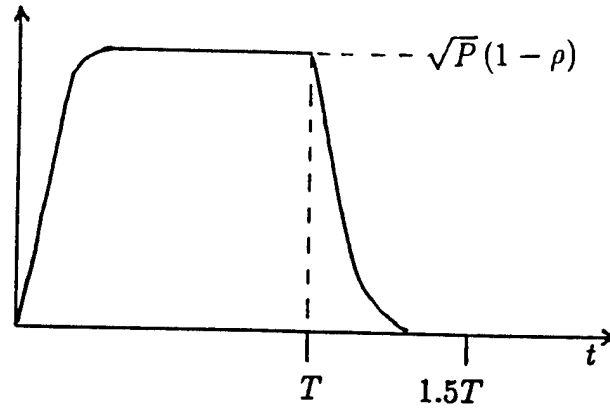
Looking at Eq. (61). We separate the terms involving the detected bit of interest in Channel 0,  $b_{0,0}$ , the trailing bits in Channel 0,  $b_{0,i}$   $i \neq 0$ , and the terms involving bits in the other channels,  $b_{k,\ell}$ ,  $k \neq 0$

$$\begin{aligned}
s(t) = r_0(t) = \underbrace{\sqrt{P} (1-\rho)}_K & \left[ \underbrace{\sum_{g=0}^{\infty} \rho^g b_{0,0} p_T \left( t - \frac{g}{\beta} \right)}_{s_B(t)} \right. \\
& + \underbrace{\sum_{i=-L_0}^{-1} \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left( t - \frac{g}{\beta} - iT \right)}_{s_{ISI}(t)} \\
& + \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left( t - \frac{g}{\beta} - \ell T \right)}_{s_{ACI}(t)} \left. \right] \quad (62)
\end{aligned}$$

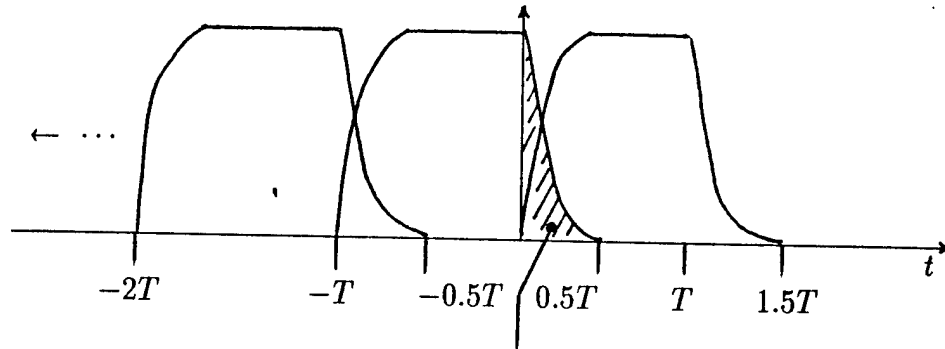
where  $s_B(t)$  is the desired signal,  $s_{ISI}(t)$  is the Intersymbol Interference (ISI) signal, and  $s_{ACI}(t)$  is the Adjacent Channel Interference (ACI) signal. Using this compact notation, we have

$$s(t) = r_0(t) = K (s_B(t) + s_{ISI}(t) + s_{ACI}(t)) \quad (63)$$

Sketching  $s_B(t)$  for  $b_{0,0} = 1$



We can also sketch the intersymbol interference due to the trailing pulses in Channel 0



ISI contributions to the pulse of interest

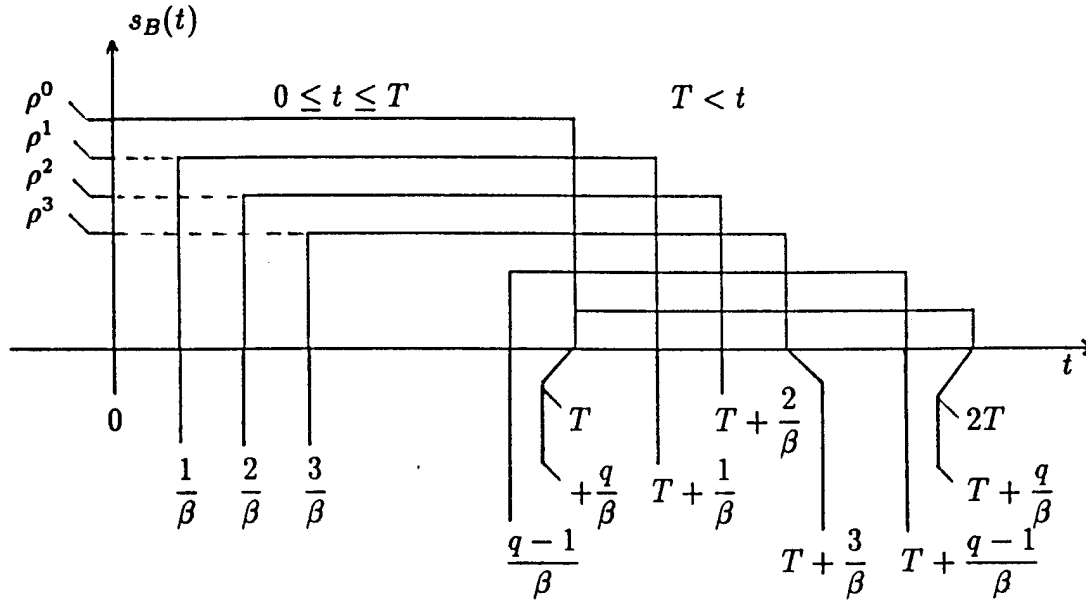
Now let us assume in one pulse interval  $T$  that

$$\frac{T}{\frac{1}{\beta}} = q \quad (64)$$

where  $1/\beta$  divides evenly into the period  $T$  making  $q$  an integer quantity. We are only interested in the detection interval  $0 \leq t \leq T$ , so we only need to evaluate  $s(t) = r_0(t)$  in the interval  $0 \leq t \leq T$ . Looking at the desired signal, which is the system response to the bit of interest  $b_{0,0}$

$$s_B(t) = \sum_{g=0}^{\infty} \rho^g b_{0,0} p_T \left( t - \frac{g}{\beta} \right), \quad \text{for } 0 \leq t < \infty \quad (65)$$

Assuming  $b_{0,0} = 1$ ,  $s_B(t)$  appears as



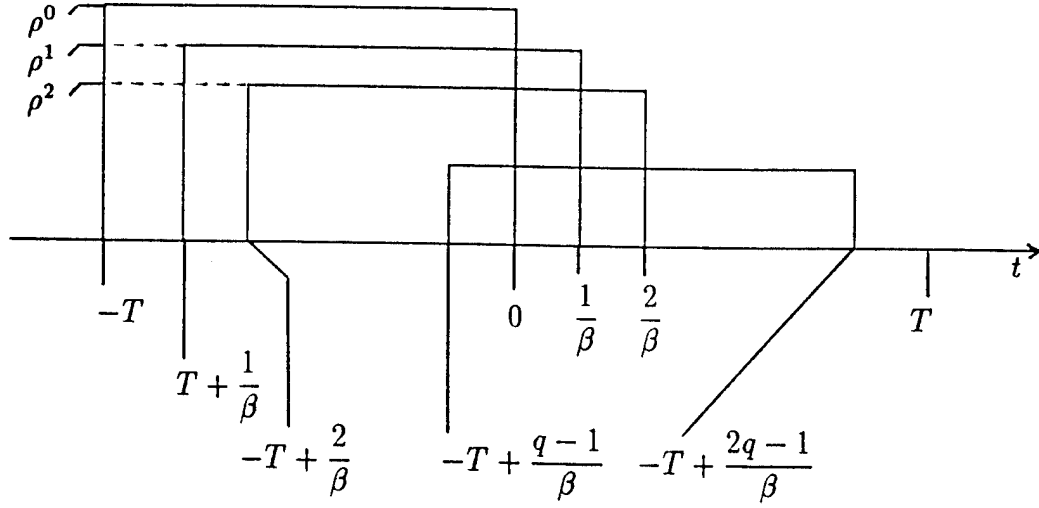
We can see that, during the interval  $0 \leq t \leq T$ , the response is a sum of scaled, shifted rectangular pulses. Therefore

$$s_B(t) = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left( t - \frac{g}{\beta} \right) \right), \quad \text{for } 0 \leq t \leq T \quad (66)$$

We now direct our efforts towards developing an expression for  $s_{ISI}(t)$  during the interval  $0 \leq t \leq T$ . We begin by examining the ISI term for  $i = -1$  in Eq. (62), denoted by  $-1ISI$ .

$$-1ISI = \sum_{g=0}^{\infty} \rho^g b_{0,-1} p_T \left( t - \frac{g}{\beta} + 1T \right), \quad \text{for } -T \leq t < \infty \quad (67)$$

By examining the sketch of  $-1ISI$



we see that  $-1ISI$  will also be a sum of shifted, scaled rectangular pulses for  $0 \leq t \leq T$ . The  $-1ISI$  expression will have two parts, the first of which is

$$-1ISI(1) = \sum_{g=1}^{q-1} \rho^g b_{0,-1pT} \left( \frac{\beta T(t)}{g} \right), \quad \text{for } 0 \leq t \leq T \quad (68)$$

But with the  $q^{\text{th}}$  shift we no longer sum scaled pulses beginning at  $t = 0$  and ending at  $1/\beta, 2/\beta, \dots$ , but now we sum pulses which progressively scale and begin at  $1/\beta, 2/\beta, \dots$ , and end at  $t = T$ . This second part of the expression looks much like the expression for  $s_B(t)$  during the interval  $0 \leq t \leq T$  [Eq. (66)]. Thus, we may write the second part of  $-1ISI$

$$-1ISI(2) = \sum_{g=q}^{2q-1} \rho^g b_{0,-1pT} \left( \frac{T}{\left( T - \left( \frac{g-q}{\beta} \right) \right)} \left( t - \left( \frac{g-q}{\beta} \right) \right) \right), \quad \text{for } 0 \leq t \leq T \quad (69)$$

Then, since  $-1ISI = -1ISI(1) + -1ISI(2)$  for  $0 \leq t \leq T$  we have

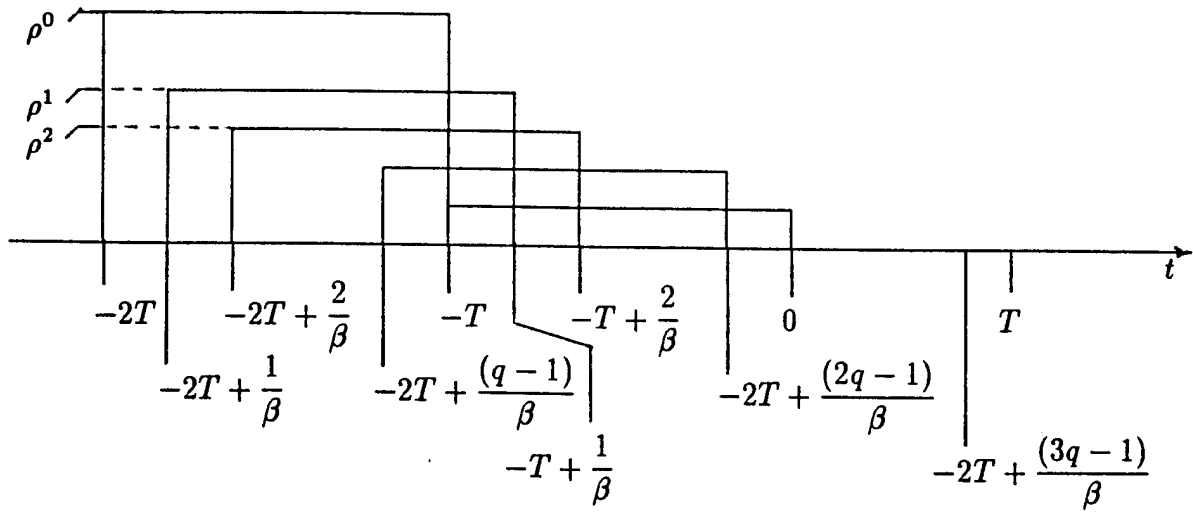
$$-1ISI = \sum_{g=1}^{q-1} \rho^g b_{0,-1pT} \left( \frac{\beta T(t)}{g} \right)$$

$$+ \sum_{g=q}^{2q-1} \rho^g b_{0,-1} p_T \left( \frac{T}{\left(T - \left(\frac{g-q}{\beta}\right)\right)} \left(t - \left(\frac{g-q}{\beta}\right)\right) \right), \quad \text{for } 0 \leq t \leq T \quad (70)$$

Examine the  $ISI$  term for  $i = -2$  in Eq. (62),  $-2ISI$

$$-2ISI = \sum_{g=0}^{\infty} b_{0,-2} \rho^g p_T \left( t - \frac{g}{\beta} + 2T \right), \quad \text{for } -2T \leq t < \infty \quad (71)$$

We sketch  $-2ISI$



By analyzing this sketch, we can obtain

$$-2ISI(1) = \sum_{g=q+1}^{2q-1} \rho^g b_{0,-2} p_T \left( \frac{\beta T(t)}{(g-q)} \right), \quad \text{for } 0 \leq t \leq T \quad (72)$$

and

$$-2ISI(2) = \sum_{g=2q}^{3q-1} \rho^g b_{0,-2} p_T \left( \frac{T}{\left(T - \left(\frac{g-2q}{\beta}\right)\right)} \left(t - \left(\frac{g-2q}{\beta}\right)\right) \right), \quad \text{for } 0 \leq t \leq T \quad (73)$$

*Note:* These rectangular pulses shift into the interval  $0 \leq t \leq T$  on the  $2q^{\text{th}}$  shift.

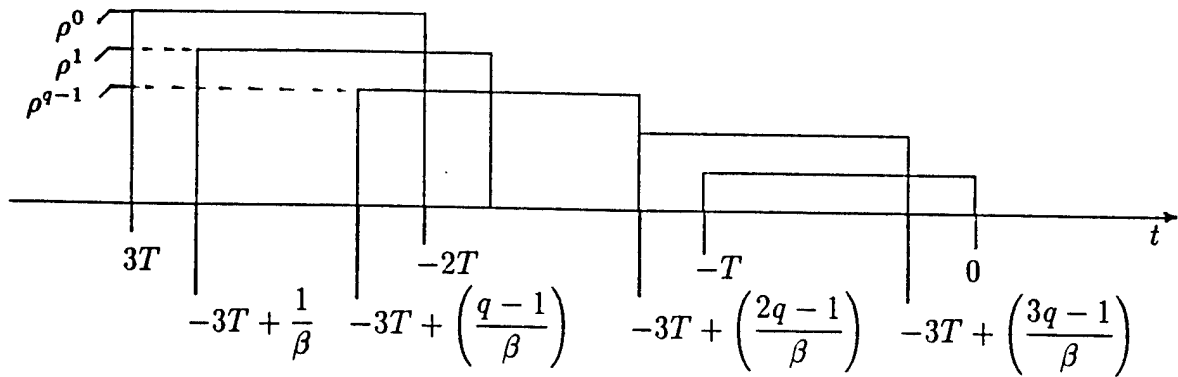
Similar to  $-1ISI$ :  $-2ISI = -2ISI(1) + -2ISI(2)$  for the interval  $0 \leq t \leq T$

$$-2ISI = \sum_{g=q+1}^{2q-1} \rho^g b_{0,-2pT} \left( \frac{\beta T(t)}{g-q} \right) + \sum_{g=2q}^{3q-1} \rho^g b_{0,-2pT} \left( \frac{T}{\left( T - \left( \frac{g-2q}{\beta} \right) \right)} \left( t - \left( \frac{g-2q}{\beta} \right) \right) \right), \quad \text{for } 0 \leq t \leq T \quad (74)$$

We now examine the  $-3ISI$  term to ensure recognition of a pattern, if any, that will allow us to write an expression for  $s_{ISI}(t)$  for  $0 \leq t \leq T$ .

$$-3ISI = \sum_{g=0}^{\infty} \rho^g b_{0,-3pT} \left( t - \frac{g}{\beta} + 3T \right), \quad \text{for } -3T \leq t < \infty \quad (75)$$

From the sketch of  $-3ISI$



we may write directly

$$-3ISI = \sum_{g=2q+1}^{3q-1} \rho^g b_{0,-3pT} \left( \frac{\beta T(t)}{g-2q} \right) + \sum_{g=3q}^{4q-1} \rho^g b_{0,-3pT} \left( \frac{T}{\left( T - \left( \frac{g-3q}{\beta} \right) \right)} \left( t - \left( \frac{g-3q}{\beta} \right) \right) \right), \quad \text{for } 0 \leq t \leq T \quad (76)$$

Then from Eqs. (70), (74), (76), the expressions for the  $-1ISI$ ,  $-2ISI$ , and  $-3ISI$  pulses, we are ready to use the pattern developed to write an expression for the ISI

between  $0 \leq t \leq T$ . Recalling

$$s_{ISI}(t) = \sum_{i=-L_0}^{-1} \sum_{g=0}^{\infty} \rho^g b_{0,i} p_T \left( t - \frac{g}{\beta} - iT \right), \quad \text{for } -L_0 T \leq t < \infty \quad (77)$$

we may write directly from the pattern developed

$$s_{ISI}(t) = \sum_{i=-L_0}^{-1} \left[ \sum_{g=-(1+i)q+1}^{(-iq)-1} \rho^g b_{0,i} p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) + \sum_{g=-iq}^{-(i-1)q-1} \rho^g b_{0,i} p_T \left( \frac{T}{T - \left( \frac{g+iq}{\beta} \right)} \left( t - \left( \frac{g+iq}{\beta} \right) \right) \right) \right], \quad \text{for } 0 \leq t \leq T \quad (78)$$

We now have expressions for  $s_B(t)$  and  $s_{ISI}(t)$ , during the interval  $0 \leq t \leq T$ . We have only the ACI term  $s_{ACI}(t)$  to calculate

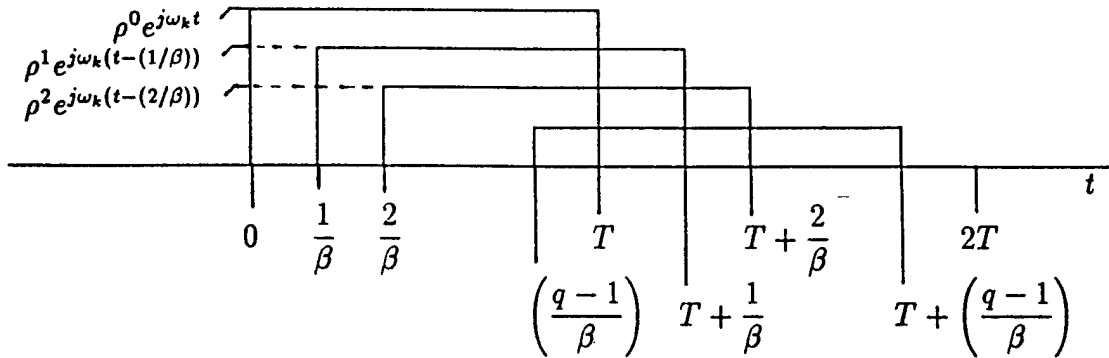
$$s_{ACI}(t) = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \underbrace{\sum_{\ell=-L}^0 \sum_{g=0}^{\infty} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left( t - \frac{g}{\beta} - \ell T \right)}_{*} \quad \text{for } -LT \leq t < \infty \quad (79)$$

\* *Note:* This portion of  $s_{ACI}(t)$  is almost exactly like the ISI term with which we just worked [see Eq. (62)]. The only difference is the complex exponential factor.

Looking at the  $0^{\text{th}}$  term of the ACI in the  $k^{\text{th}}$  channel

$$0ACI = \sum_{g=0}^{\infty} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left( t - \frac{g}{\beta} \right), \quad \text{for } 0 \leq t < \infty \quad (80)$$

we sketch  $0ACI$



We see, that in a manner similar to the  $s_B(t)$  signal, the pulses add as before, but now the amplitude of the pulses is modulated by  $\rho^g e^{j\omega_k(t-(g/\beta))}$  versus just  $\rho^g$  before. Then

$$0ACI = \sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right), \quad \text{for } 0 \leq t \leq T \quad (81)$$

The  $-1ACI$  will be similar to the  $-1ISI$  term [see Eq. (70)]. However, now we not only have  $\rho^g$  but also the complex exponential gated by the pulses in the summation.

$$\begin{aligned} -1ACI &= \sum_{g=1}^{q-1} \rho^g b_{k,-1} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{\beta T(t)}{g} \right) \\ &+ \sum_{g=q}^{2q-1} \rho^g b_{k,-1} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{T}{\left(T - \left(\frac{g-q}{\beta}\right)\right)} \left(t - \left(\frac{g-q}{\beta}\right)\right) \right), \quad \text{for } 0 \leq t \leq T \end{aligned} \quad (82)$$

The derivation of  $-2ACI$  and  $-3ACI$  will proceed in a similar manner as  $-2ISI$  and  $-3ISI$  [Eqs. (74) and (76)]. We see that two different type pulses gate the product of  $\rho^g$  and a complex exponential term. These gated terms are then summed over the period 0 to  $T$ . We then use our expressions for  $0ACI$ ,  $-1ACI$ , ... to write an expression for  $s_{ACI}(t)$  during the interval  $0 \leq t \leq T$  in a manner similar to the  $s_{ISI}(t)$  expression. We may now write the final expression for the ACI

$$\begin{aligned} s_{ACI}(t) &= \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \left[ \sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right] \\ &+ \sum_{\ell=-L}^{-1} \left[ \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} p_T \left( \frac{\beta T(t)}{g + (1+\ell)q} \right) \right] \\ &+ \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} \times \dots \end{aligned}$$



$$p_T \left( \frac{T}{\left(T - \left(\frac{g + \ell q}{\beta}\right)\right)} \left( t - \left(\frac{g + \ell q}{\beta}\right) \right) \right) \right], \quad \text{for } 0 \leq t \leq T \quad (83)$$

We have now derived  $s_B(t)$ ,  $s_{ISI}(t)$ , and  $s_{ACI}(t)$  during the interval  $0 \leq t \leq T$ . We now develop the expressions needed to compute  $|s(t)|^2$  and  $\int_0^T |s(t)|^2 dt$ . Substituting the expressions for  $s_B(t)$ ,  $s_{ISI}(t)$ , and  $s_{ACI}(t)$  for the interval  $0 \leq t \leq T$  into Eq. (63) yields  $s(t)$  in the interval  $0 \leq t \leq T$

$$s(t) = \underbrace{\sqrt{P}(1 - \rho)}_K \underbrace{[s_B(t) + s_{ISI}(t) + s_{ACI}(t)]}_{s'(t)}, \quad \text{for } 0 \leq t \leq T \quad (84)$$

where  $s_B(t)$ ,  $s_{ISI}(t)$ , and  $s_{ACI}(t)$  are described by Eqs. (66), (78), and (83) respectively. Then

$$|s(t)|^2 = K^2 s'(t) s'(t)^* \quad (85)$$

Dropping the  $(t)$  notation for convenience in the three terms of  $s'(t)$  and noting that complex conjugation is a linear operator

$$s'(t)^* = s_B^* + s_{ISI}^* + s_{ACI}^* \quad (86)$$

So

$$s'(t) s'(t)^* = (s_B + s_{ISI} + s_{ACI})(s_B^* + s_{ISI}^* + s_{ACI}^*) \quad (87)$$

Multiplication yields

$$\begin{aligned} s'(t) s'(t)^* &= s_B s_B^* + s_B s_{ISI}^* + s_B s_{ACI}^* \\ &\quad + s_{ISI} s_B^* + s_{ISI} s_{ISI}^* + s_{ISI} s_{ACI}^* \\ &\quad + s_{ACI} s_B^* + s_{ACI} s_{ISI}^* + s_{ACI} s_{ACI}^* \end{aligned} \quad (88)$$

Rearranging and simplifying yields

$$|s(t)|^2 = K^2 s'(t) s'(t)^* = K^2 \times \left[ \underbrace{s_B s_B^*}_{|s_B|^2} + \underbrace{s_{ISI} s_{ISI}^*}_{|s_{ISI}|^2} + \underbrace{s_{ACI} s_{ACI}^*}_{|s_{ACI}|^2} \right]$$

$$\begin{aligned}
& + \underbrace{s_B s_{ISI}^* + s_{ISI} s_B^*}_{2\text{Re}[s_B s_{ISI}^*]} = 2\text{Re}[s_B s_{ISI}^*] \\
& + \underbrace{s_B s_{ACI}^* + s_{ACI} s_B^*}_{2\text{Re}[s_B^* s_{ACI}]} = 2\text{Re}[s_B^* s_{ACI}] \\
& + \underbrace{s_{ISI} s_{ACI}^* + s_{ACI} s_{ISI}^*}_{2\text{Re}[s_{ISI}^* s_{ACI}]} = 2\text{Re}[s_{ISI}^* s_{ACI}] \quad (89)
\end{aligned}$$

We will integrate each of these terms over 0 to  $T$  to compute  $\int_0^T |s(t)|^2 dt$ . To avoid confusion, we must realize that constant  $K^2$  must be carried through to the final calculation of all quantities. Recalling Eq. (66)

$$s_B = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \quad (90)$$

We know  $b_{0,i} \in (0, 1)$ ,  $\rho^g$  is real, and  $p_T(\cdot)$  is also real. Then

$$s_B = s_B^* \quad (91)$$

and

$$|s_B|^2 = \left( \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right)^2 \quad (92)$$

Factoring  $b_{0,0}$  as it is a constant to the summation, we obtain

$$|s_B|^2 = b_{0,0}^2 \left( \sum_{g=0}^{q-1} \rho^g p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right)^2 \quad (93)$$

Performing the squaring operation yields

$$|s_B|^2 = b_{0,0}^2 \left( \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} p_T \left( \frac{T}{T - \frac{g}{\beta}} \left( t - \frac{g}{\beta} \right) \right) p_T \left( \frac{T}{T - \frac{m}{\beta}} \left( t - \frac{m}{\beta} \right) \right) \right) \quad (94)$$

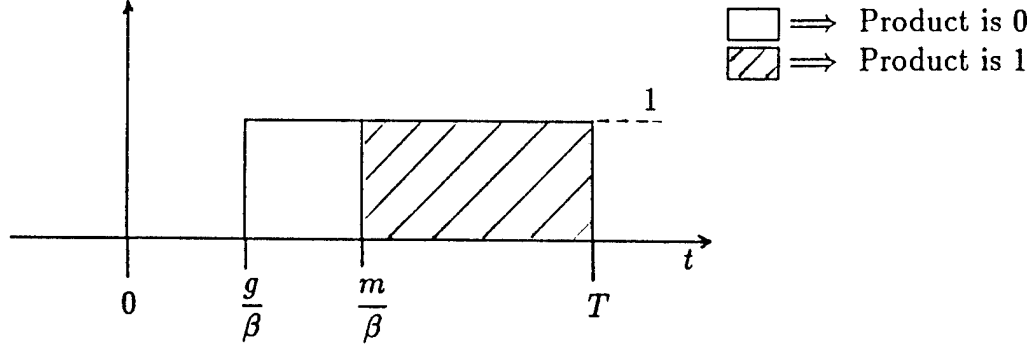
Then

$$K^2 |s_B|^2 = P(1 - \rho)^2 b_{0,0}^2 \left[ \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} p_T \left( \frac{T}{T - \frac{g}{\beta}} \left( t - \frac{g}{\beta} \right) \right) \times \dots \right. \\ \left. p_T \left( \frac{T}{T - \frac{m}{\beta}} \left( t - \frac{m}{\beta} \right) \right) \right], \quad \text{for } 0 \leq t \leq T \quad (95)$$

Now we want to compute  $\int_0^T K^2 |s_B|^2 dt$  [see Eq. (89)].

$$\int_0^T K^2 |s_B|^2 dt = P(1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \times \dots \\ \underbrace{\int_0^T p_T \left( \frac{T}{T - \frac{g}{\beta}} \left( t - \frac{g}{\beta} \right) \right) p_T \left( \frac{T}{T - \frac{m}{\beta}} \left( t - \frac{m}{\beta} \right) \right) dt}_{\eta} \quad (96)$$

Looking at Eq. (96), we have interchanged the order of integration and summation. As the pulse functions are the only functions dependent on time in the expression, we look at the product of the two unit amplitude pulse functions in  $\eta$



We have arbitrarily assumed that  $m > g$ , but our logic would also work if we assumed  $g > m$ . The product of two pulses is 1 for  $m/\beta \leq t \leq T$  and 0 otherwise. Thus, we see that the integral of the product of the two pulse functions in  $\eta$  will be a square area of height  $1 \times (T - \text{largest of } g/\beta \text{ or } m/\beta)$ . Thus

$$\int_0^T K^2 |s_B|^2 dt = P(1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left( T - \left( \frac{\max(g, m)}{\beta} \right) \right), \quad \text{for } 0 \leq t \leq T \quad (97)$$

where  $\max(g, m)$  is defined by

$\max(x_1, x_2)$ : Choose largest of  $x_1$  or  $x_2$ , which are both positive.  
If  $x_1 = x_2$ , then  $\max(x_1, x_2) = x_1 = x_2$ .

We now need to compute  $\int_0^T K^2 |s_{ISI}|^2 dt$ . We see from Eq. (78) that  $s_{ISI}$  is real and not complex since  $b_{0,i} \in \{0, 1\}$ . Thus

$$s_{ISI} = s_{ISI}^* \quad \text{and} \quad |s_{ISI}|^2 = s_{ISI}^2 \quad (98)$$

$$|s_{ISI}|^2 = \left[ \underbrace{\sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-i)q-1} \rho^g b_{0,i} p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right)}_a + \underbrace{\sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \rho^g b_{0,i} p_T \left( \frac{T}{\left( T - \left( \frac{g+iq}{\beta} \right) \right) \left( t - \left( \frac{g+iq}{\beta} \right) \right)} \right)}_b \right]^2 \quad (99)$$

To perform the squaring we set up another version of  $a + b$  with the  $g$ -index changed to  $m$  and the  $i$ -index changed to  $r$  to account for cross-product terms. We call this new representation  $c + d$ . Thus

$$|s_{ISI}|^2 = (a + b)(c + d) = \underbrace{ac}_{\mathcal{A}} + \underbrace{ad}_{\mathcal{B}} + \underbrace{bc}_{\mathcal{C}} + \underbrace{bd}_{\mathcal{D}}$$

Performing the multiplications yields

$$\begin{aligned} \mathcal{A} = ac &= \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \\ & p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{\beta T(t)}{m + (1+r)q} \right) \end{aligned} \quad (100)$$

$$\begin{aligned} \mathcal{B} = ad &= \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \\ & p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{T \left( t - \left( \frac{m + rq}{\beta} \right) \right)}{\left( T - \left( \frac{m + rq}{\beta} \right) \right)} \right) \end{aligned} \quad (101)$$

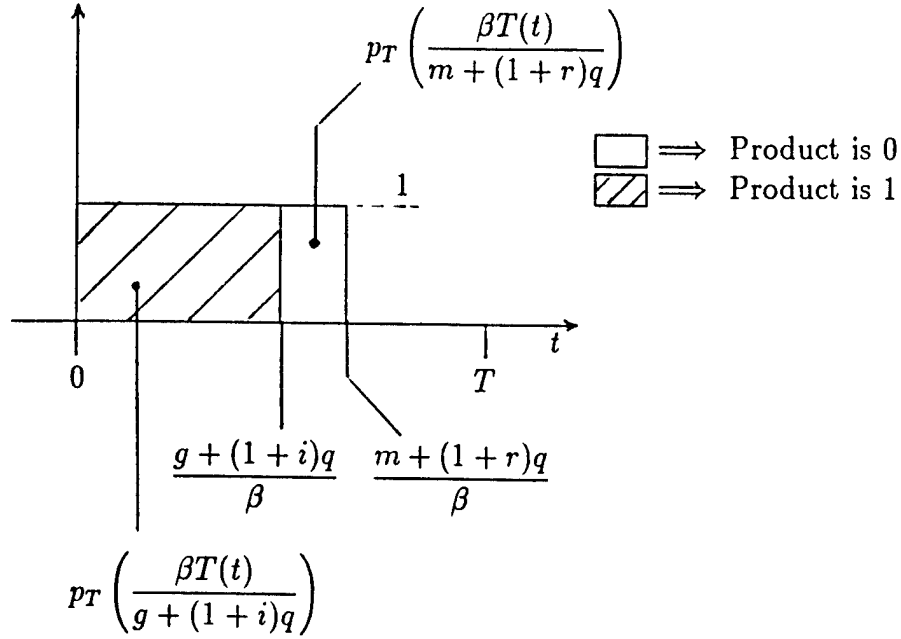
$$\begin{aligned} \mathcal{C} = bc &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \\ & p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{\beta T(t)}{m + (1+r)q} \right) \end{aligned} \quad (102)$$

$$\begin{aligned} \mathcal{D} = bd &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \\ & p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{T \left( t - \left( \frac{m + rq}{\beta} \right) \right)}{\left( T - \left( \frac{m + rq}{\beta} \right) \right)} \right) \end{aligned} \quad (103)$$

Integrating over the interval of interest:  $\int_0^T K^2 |s_{ISI}|^2 dt = \dots$

$$K^2 \int_0^T |s_{ISI}|^2 dt = K^2 \left[ \int_0^T \mathcal{A} dt + \int_0^T \mathcal{B} dt \int_0^T \mathcal{C} dt + \int_0^T \mathcal{D} dt \right] \quad (104)$$

The equations for  $\mathcal{A}$ ,  $\mathcal{B}$ ,  $\mathcal{C}$ , and  $\mathcal{D}$  are Eqs. (100)–(103), respectively. We now analyze term  $\mathcal{A}$  [Eq. (100)] by sketching the multiplication of the pulses



Again, we have arbitrarily assumed one pulse lasts longer than the other. Since the height of each scaled pulse is 1, we see that the area under the product is 1 times the minimum of

$$\frac{q + (1+i)q}{\beta} \quad \text{or} \quad \frac{m + (1+r)q}{\beta}$$

We can apply the same logic as used before to arrive at the value of the term

$$K^2 \int_0^T \mathcal{A} dt$$

Then

$$K^2 \int_0^T \mathcal{A} dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0 \quad (105)$$

where

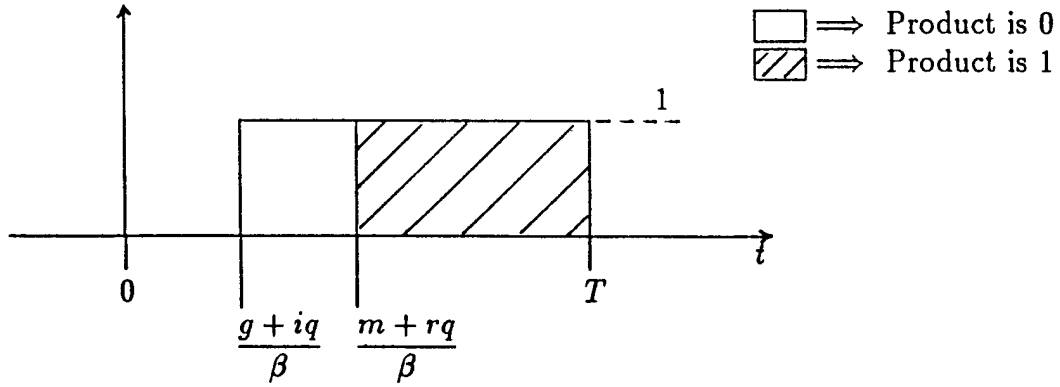
$$\varphi_0 = \rho^{g+m} b_{0,i} b_{0,r} \left( \frac{\min(g + (1+i)q, m + (1+r)q)}{\beta} \right)$$

where

$\min(g + (1+i)q, m + (1+r)q)$  is defined by

$\min(x_1, x_2)$ : Choose smallest of  $x_1$  or  $x_2$ , which are both positive.  
If  $x_1 = x_2$ , then  $\min(x_1, x_2) = x_1 = x_2$ .

We now compute  $K^2 \int_0^T \mathcal{D} dt$ . Looking at the expression for  $\mathcal{D}$  [Eq. (103)], we sketch the product of the pulses, arbitrarily assuming  $m + rq > g + iq$



We see the product of pulses only exists from the maximum of

$$\frac{g+iq}{\beta} \quad \text{or} \quad \frac{m+rq}{\beta}$$

Thus, the integral of the product of the pulses will be

$$T - \frac{\max(g+iq, m+rq)}{\beta}$$

Then

$$K^2 \int_0^T \mathcal{D} dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \left[ T - \left( \frac{\max(g+iq, m+rq)}{\beta} \right) \right] \quad (106)$$

Looking at the expression for  $\mathcal{B}$  [Eq. (101)]

$$\begin{aligned} \mathcal{B} = & \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \\ & p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{T \left( t - \left( \frac{m+rq}{\beta} \right) \right)}{\left( T - \left( \frac{m+rq}{\beta} \right) \right)} \right) \end{aligned} \quad (107)$$

We note that  $m$ ,  $g$ ,  $i$ , and  $r$  are simply dummy variables and may be interchanged, giving

$$\begin{aligned} \mathcal{B} = & \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,r} b_{0,i} \times \dots \\ & p_T \left( \frac{\beta T(t)}{m + (1+r)q} \right) p_T \left( \frac{T \left( t - \left( \frac{g+iq}{\beta} \right) \right)}{\left( T - \left( \frac{g+iq}{\beta} \right) \right)} \right) \end{aligned} \quad (108)$$

If we look at the expression above Eq. (108), we see that  $\mathcal{B}$  is identical to  $\mathcal{C}$  [Eq. (102)] in every way except the order in which the summations appear. The order of the summations can be rearranged because the terms inside of the summations are completely separable in relation to the sets  $\{m, r\}$ ,  $\{i, g\}$ . Thus, we can rearrange the order of the summations and conclude

$$\mathcal{B} = \mathcal{C} \quad (109)$$

and that

$$K^2 \int_0^T \mathcal{B} dt + K^2 \int_0^T \mathcal{C} dt = 2K^2 \int_0^T \mathcal{B} dt = 2K^2 \int_0^T \mathcal{C} dt \quad (110)$$

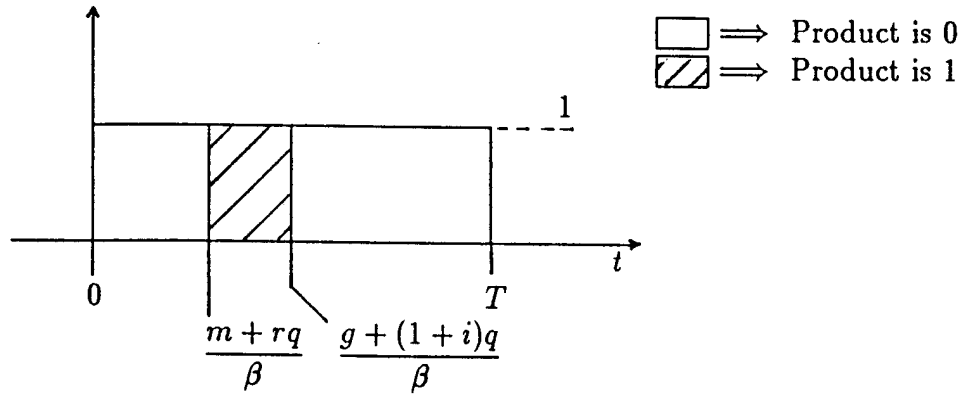
We choose to integrate twice the value of  $\mathcal{B}$  [Eq. (101)]. Interchanging the order of integration and summation yields

$$2K^2 \int_0^T \mathcal{B} dt = 2K^2 \left( \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \right)$$



$$\int_0^T p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{T \left( t - \left( \frac{m + rq}{\beta} \right) \right)}{\left( T - \left( \frac{m + rq}{\beta} \right) \right)} \right) dt \quad (111)$$

Sketching the product of the two pulses, we see the product will exist for  $m + rq < g + (1+i)q$  and will be zero otherwise.



We create a "gating" function to account for this

$$\int_0^T p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{T \left( t - \left( \frac{m + rq}{\beta} \right) \right)}{\left( T - \left( \frac{m + rq}{\beta} \right) \right)} \right) dt =$$

$$G_1(g, i, m, r) = \left\{ \begin{array}{ll} \frac{1}{\beta} [(g + (1+i)q) - (m + rq)], & \text{for } m + rq < g + (1+i)q \\ 0, & \text{otherwise} \end{array} \right\} \quad (112)$$

Then finally

$$\begin{aligned} 2K^2 \int_0^T B dt &= K^2 \int_0^T B dt + K^2 \int_0^T C dt \\ &= 2P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-i)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g, i, m, r) \end{aligned} \quad (113)$$

Finally, combining the results of Eqs. (105), (106), and (113) we have

$$K^2 \int_0^T s_{ISI} s_{ISI}^* dt = K^2 \int_0^T |s_{ISI}|^2 dt = K^2 \left[ \int_0^T \mathcal{A} dt + \int_0^T \mathcal{B} dt \int_0^T \mathcal{C} dt + \int_0^T \mathcal{D} dt \right] \quad (114)$$

where

$$K^2 \int_0^T \mathcal{A} dt = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0$$

in which

$$\begin{aligned} \varphi_0 &= \rho^{g+m} b_{0,i} b_{0,r} \left( \frac{\min(g + (1+i)q, m + (1+r)q)}{\beta} \right) \\ K^2 \int_0^T \mathcal{B} dt + K^2 \int_0^T \mathcal{C} dt &= 2K^2 \int_0^T \mathcal{B} dt \\ &= 2P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g, i, m, r) \end{aligned}$$

and

$$\begin{aligned} K^2 \int_0^T \mathcal{D} dt &= P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \times \dots \\ &\quad \left[ T - \left( \frac{\max(g + iq, m + rq)}{\beta} \right) \right] \end{aligned}$$

Now we must compute  $|s_{ACI}|^2$  and  $K^2 \int_0^T |s_{ACI}|^2 dt$ . We see that we have a complex  $b_{k,\ell}$  as  $b_{k,\ell} \in \{0, e^{j\phi_k}\}$ , as well as a complex exponential  $e^{j\omega_k(t-(g/\beta))}$  [see Eq. (83)].

We begin by noting that the complex conjugate operator has the following properties

$$(uv)^* = u^* v^*$$

$$(u+v)^* = u^* + v^*$$

Taking the complex conjugate of Eq. (83)

$$s_{ACI}^* = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \left( \left[ \sum_{g=0}^{q-1} \rho^g b_{k,0}^* e^{-j\omega_k(t-(g/\beta))} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \right] \right)$$

$$\begin{aligned}
& + \sum_{\ell=-L}^{-1} \left[ \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell}^* e^{-j\omega_k(t-(g/\beta))} p_T \left( \frac{\beta T(t)}{g + (1+\ell)q} \right) \right. \\
& \left. + \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell}^* e^{-j\omega_k(t-(g/\beta))} p_T \left( \frac{T \left( t - \left( \frac{g+\ell q}{\beta} \right) \right)}{\left( T - \left( \frac{g+\ell q}{\beta} \right) \right)} \right) \right] \quad (115)
\end{aligned}$$

Letting

$$a = p_T \left( \frac{T}{\left( T - \frac{g}{\beta} \right)} \left( t - \frac{g}{\beta} \right) \right) \quad (116)$$

$$a' = p_T \left( \frac{T}{\left( T - \frac{m}{\beta} \right)} \left( t - \frac{m}{\beta} \right) \right) \quad (117)$$

and

$$c = p_T \left( \frac{\beta T(t)}{g + (1+\ell)q} \right) \quad (118)$$

$$c' = p_T \left( \frac{\beta T(t)}{m + (1+r)q} \right) \quad (119)$$

$$d = p_T \left( \frac{T \left( t - \left( \frac{g+\ell q}{\beta} \right) \right)}{\left( T - \left( \frac{g+\ell q}{\beta} \right) \right)} \right) \quad (120)$$

$$d' = p_T \left( \frac{T \left( t - \left( \frac{m+r q}{\beta} \right) \right)}{\left( T - \left( \frac{m+r q}{\beta} \right) \right)} \right) \quad (121)$$

Then, with a change of indices, we can express  $s_{ACI}^*$  as

$$\begin{aligned}
s_{ACI}^* &= \underbrace{\sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^m b_{n,0}^* e^{-j\omega_n(t-(m/\beta))} a'}_{A^*}
\end{aligned}$$

$$\begin{aligned}
& + \underbrace{\sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^m b_{n,r}^* e^{-j\omega_n(t-(m/\beta))} c'}_{B^*} \\
& + \underbrace{\sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^m b_{n,r}^* e^{-j\omega_n(t-(m/\beta))} d'}_{C^*}
\end{aligned} \tag{122}$$

Now

$$\begin{aligned}
s_{ACI} &= \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \rho^g b_{k,0} e^{j\omega_k(t-(g/\beta))} a}_{A} \\
& + \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} c}_{B} \\
& + \underbrace{\sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \rho^g b_{k,\ell} e^{j\omega_k(t-(g/\beta))} d}_{C}
\end{aligned} \tag{123}$$

Then

$$|s_{ACI}|^2 = s_{ACI} s_{ACI}^* = (A + B + C)(A^* + B^* + C^*) \tag{124}$$

Now, distribution of multiplication over addition yields

$$\begin{aligned}
|s_{ACI}|^2 &= AA^* + AB^* + AC^* + BA^* + BB^* + BC^* \\
&+ CA^* + CB^* + CC^*
\end{aligned} \tag{125}$$

Rearranging yields

$$\begin{aligned}
|s_{ACI}|^2 &= AA^* + BB^* + CC^* + 2 \left[ \frac{1}{2} (AB^* + BA^*) \right] \\
&\quad + 2 \left[ \frac{1}{2} (AC^* + CA^*) \right] + 2 \left[ \frac{1}{2} (BC^* + CB^*) \right]
\end{aligned} \tag{126}$$

Using the identity  $Re[Z] = (Z + Z^*)/2$  we have

$$\begin{aligned}
|s_{ACI}|^2 &= AA^* + BB^* + CC^* + 2Re[AC^*] \\
&\quad + 2Re[AB^*] + 2Re[BC^*]
\end{aligned} \tag{127}$$

Multiplying the terms above yields

$$AA^* = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{k,0} b_{n,0}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} aa' \tag{128}$$

$$BB^* = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \gamma \tag{129}$$

where

$$\gamma = \rho^{g+m} b_{k,\ell} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} cc'$$

$$CC^* = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \zeta \tag{130}$$

where

$$\zeta = \rho^{g+m} b_{k,\ell} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} dd'$$

To compute  $2Re[AC^*]$ , we know  $Re[\cdot]$  is a linear operator so it may be moved across the summations. After multiplication of terms A and C\* we get

$$2Re[AC^*] = 2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \kappa \tag{131}$$

where

$$\kappa = Re \left[ \rho^{g+m} b_{k,0} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \text{ad}' \right]$$

We know  $b_{k,0}$ ,  $b_{n,r}^*$ , and  $e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]}$  are complex. We will calculate  $\kappa$  in Eq. (131) using the identity

$$Re[Z] = \frac{1}{2}(Z + Z^*)$$

Since  $(uv)^* = u^*v^*$ , by extension it can be shown that

$$(uvw)^* = u^*v^*w^*$$

Also note that if  $S = a_1 + a_2 + a_3 + a_4 + \dots$  where  $a_1, a_2, a_3, a_4, \dots$  are complex, then

$$Re[S] = Re[a_1] + Re[a_2] + Re[a_3] + Re[a_4] + \dots$$

We will also use the fact that if  $z$  is complex and  $\alpha$  is constant, then

$$Re[\alpha z] = \alpha Re[z]$$

Using the facts above, we can see

$$\begin{aligned} \kappa = & \frac{1}{2} \left[ b_{k,0} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ & \left. + b_{k,0}^* b_{n,r} e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \rho^{g+m} \text{ad}' \end{aligned} \quad (132)$$

Then

$$\begin{aligned} \kappa = & \left[ b_{k,0} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ & \left. + b_{k,0}^* b_{n,r} e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \rho^{g+m} \text{ad}' \end{aligned} \quad (133)$$

with  $a$  and  $d'$  defined by Eqs. (116) and (121), respectively.

Now, we will turn to computing  $2Re[AB^*]$ . This will have five summations just like the  $2Re[AC^*]$  term above.

$$2Re[AB^*] = \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\substack{g=0 \\ g \neq 0}}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varsigma \quad (134)$$

By inspection of Eq. (123)

$$\varsigma = Re \left[ \rho^{g+m} b_{k,0} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} a c' \right]$$

After letting the  $1/2$  generated when applying  $Re[\cdot]$  operator to the argument cancel the 2 outside of the summations, we get

$$\begin{aligned} \varsigma = & \left[ b_{k,0} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ & \left. + b_{k,0}^* b_{n,r} e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \rho^{g+m} a c' \end{aligned} \quad (135)$$

With  $a$  and  $c'$  defined by Eqs. (116) and (119), respectively.

We now compute the term  $2Re[BC^*]$ . This term will have six summations. Moving the  $Re[\cdot]$  operator inside of the six summations to the argument yields the following by inspection

$$\iota = Re \left[ b_{k,\ell} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \rho^{g+m} c d' \right] \quad (136)$$

After letting the  $1/2$  generated when applying the  $Re[\cdot]$  operator in  $\iota$  [Eq. (136)] cancel the 2 outside of the summations, we can then write down the expression for  $2Re[BC^*]$

$$\begin{aligned} 2Re[BC^*] = & \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \iota \quad (137) \\ \iota = & \left[ b_{k,\ell} b_{n,r}^* e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} + b_{k,\ell}^* b_{n,r} \times \dots \right. \\ & \left. e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \rho^{g+m} c d' \end{aligned}$$

with  $c$  and  $d'$  defined by Eqs. (118) and (121).

Now we must find

$$\begin{aligned} \int_0^T K^2 |s_{ACI}|^2 dt &= K^2 \int_0^T AA^* dt + K^2 \int_0^T BB^* dt + K^2 \int_0^T CC^* dt \\ &+ K^2 \int_0^T 2\text{Re}[AC^*] dt + K^2 \int_0^T 2\text{Re}[AB^*] dt \\ &+ K^2 \int_0^T 2\text{Re}[BC^*] dt \end{aligned} \quad (138)$$

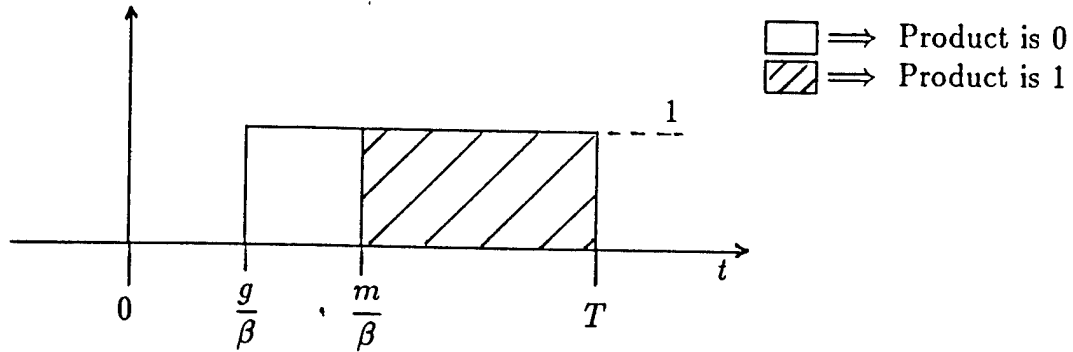
where we recall that

$$K^2 = P(1 - \rho)^2$$

In the term  $AA^*$  [Eq. (128)], we have the pulse multiplication  $aa'$  where

$$a = p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) \quad a' = p_T \left( \frac{T \left( t - \frac{m}{\beta} \right)}{\left( T - \frac{m}{\beta} \right)} \right)$$

Arbitrarily assuming  $m > g$ , we sketch the product  $a \times a'$ .



The product is 1 and exists from the maximum of  $(m/\beta, g/\beta)$  to  $T$ . Either  $g$  or  $m$  can be the largest. Hence, we will be integrating

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (139)$$

with the lower limit of integration being  $(1/\beta) \max(g, m)$ . The upper limit of integration is  $T$ . We consider two cases for Eq. (128).



**Case 1:**  $k \neq n \implies \omega_k \neq \omega_n$

When  $k \neq n$ ,  $\omega_k \neq \omega_n$  because

$$\omega_k = k\Delta\omega \quad (140)$$

where again  $\omega_k$  is the radian frequency spacing between Channel 0 and Channel  $k$ , and  $\Delta\omega$  is the uniform radian frequency spacing between adjacent channels.

$$K^2 \int_0^T \mathbf{A} \mathbf{A}^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{k,0} b_{n,0}^* \times \dots \int_{(1/\beta) \max(g,m)}^T \xi dt \quad (141)$$

Letting

$$x = j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]$$

$$dx = j[(\omega_k - \omega_n)] dt$$

then

$$\begin{aligned} \int_{(1/\beta) \max(g,m)}^T \xi dt &= \frac{1}{j(\omega_k - \omega_n)} e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big|_{(1/\beta) \max(g,m)}^T \\ &= \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ &\quad \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g,m) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \end{aligned} \quad (142)$$

**Case 2:**  $k = n \implies \omega_k = \omega_n$

Again similar logic to that used above [Eq. (140)] allows us to conclude  $\omega_k = \omega_n$ . For this case, the complex exponential  $\xi$  reduces to

$$\xi = e^{j[0 - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (143)$$

Now, since  $\omega_k = \omega_n$ , we can further reduce the complex exponential to

$$\xi = e^{j[(\omega_k/\beta)(m-g)]} \quad (144)$$

Therefore, for  $\omega_k = \omega_n$

$$\int_{(1/\beta)\max(g,m)}^T \xi dt = e^{j[(\omega_k/\beta)(m-g)]} \int_{(1/\beta)\max(g,m)}^T dt = e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g, m) \right] \quad (145)$$

Thus, we may write the expression for  $K^2 \int_0^T AA^* dt$

$$K^2 \int_0^T AA^* dt = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} \times \varphi_1 \quad (146)$$

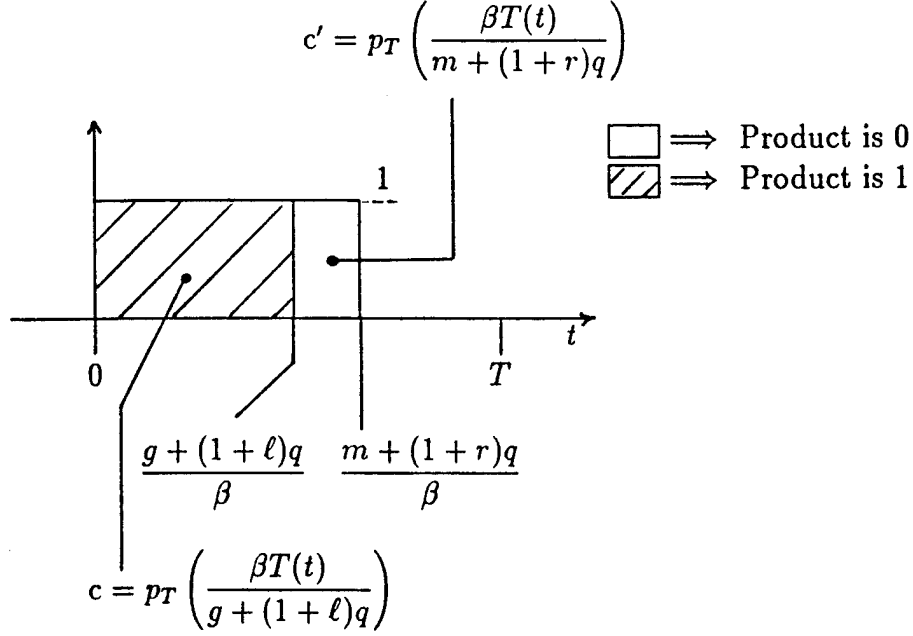
where

$$\varphi_1 = \left\{ \begin{array}{ll} b_{k,0} b_{n,0}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ \left. - e^{j[(\omega_k - \omega_n)(1/\beta)\max(g,m) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right], & \text{for } k \neq n \\ \underbrace{b_{k,0} b_{k,0}^*}_{|b_{k,0}|^2} e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g, m) \right], & \text{for } k = n \end{array} \right\}$$

Now we turn to the  $BB^*$  term [Eq. (129)]. We have the pulse multiplication  $cc'$  where

$$c = p_T \left( \frac{\beta T(t)}{g + (1 + \ell)q} \right) \quad c' = p_T \left( \frac{\beta T}{m + (1 + r)q} \right)$$

Arbitrarily assuming  $m + (1 + r)q > g + (1 + \ell)q$ , we sketch the product  $c \times c'$ .



The product exists and is 1 from 0 to minimum of

$$\left( \frac{g + (1+l)q}{\beta}, \frac{m + (1+r)q}{\beta} \right)$$

Hence, we will be integrating the same complex exponential  $\xi$  [Eq. (139)] as for the  $AA^*$  term [Eq. (128)], but the upper limit of integration will be

$$\frac{1}{\beta} \min(g + (1+l)q, m + (1+r)q)$$

while the lower limit is 0. We again consider two cases.

**Case 1:**  $k \neq n \Rightarrow \omega_k \neq \omega_n$

After interchanging the order of integration and summation, we will be integrating [see Eq. (129)]

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (147)$$

Then

$$\int_0^{(1/\beta) \min(g + (1+l)q, m + (1+r)q)} \xi dt$$

$$\begin{aligned}
&= \frac{1}{j(\omega_k - \omega_n)} e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big|_0^{(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)} \\
&= \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)(1/\beta) \min(g + (1+\ell)q, m + (1+r)q) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\
&\quad \left. - e^{j[-\omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \tag{148}
\end{aligned}$$

Then, factoring the result yields

$$\begin{aligned}
&\int_0^{(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)} \xi dt = \\
&\frac{1}{j(\omega_k - \omega_n)} e^{j[-\omega_k(g/\beta) + \omega_n(m/\beta)]} \left[ e^{j[(\omega_k - \omega_n)(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)]} - 1 \right] \tag{149}
\end{aligned}$$

**Case 2:**  $k = n \implies \omega_k = \omega_n$

This is similar to the AA\* term for  $\omega_k = \omega_n$  [see Eq. (144)]

$$\xi = e^{j[(\omega_k/\beta)(m-g)]} \tag{150}$$

and

$$\begin{aligned}
&\int_0^{(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)} \xi dt = e^{j[(\omega_k/\beta)(m-g)]} \int_0^{(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)} dt \\
&= e^{j[(\omega_k/\beta)(m-g)]} \left[ \frac{1}{\beta} \min(g + (1+\ell)q, m + (1+r)q) - 0 \right] \tag{151}
\end{aligned}$$

Thus, we may write the expression for  $K^2 \int_0^T \text{BB}^* dt$

$$\begin{aligned}
K^2 \int_0^T \text{BB}^* dt &= P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} \times \varphi_2 \\
&\tag{152}
\end{aligned}$$

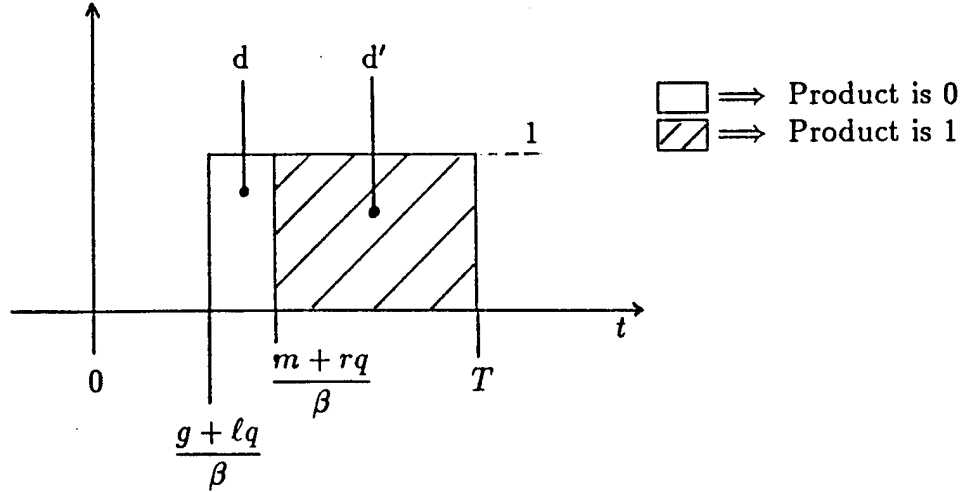
where

$$\varphi_2 = \begin{cases} b_{k,\ell} b_{n,r}^* \left[ \frac{e^{j[-\omega_k(g/\beta) + \omega_n(m/\beta)]}}{j(\omega_k - \omega_n)} \left( e^{j[(\omega_k - \omega_n)(1/\beta) \min(g + (1+\ell)q, m + (1+r)q)]} - 1 \right) \right], & \text{for } k \neq n \\ = b_{k,\ell} b_{k,r}^* \left( e^{j[(\omega_k/\beta)(m-g)]} \left[ \frac{1}{\beta} \min(g + (1+\ell)q, m + (1+r)q) \right] \right), & \text{for } k = n \end{cases}$$

Now we integrate the CC\* term [Eq. (130)]. We have the pulse multiplication  $dd'$  where

$$d = p_T \left( \frac{T \left( t - \left( \frac{g + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{g + \ell q}{\beta} \right) \right)} \right) \quad d' = p_T \left( \frac{T \left( t - \left( \frac{m + rq}{\beta} \right) \right)}{\left( T - \left( \frac{m + rq}{\beta} \right) \right)} \right)$$

Looking at the pulse multiplication graphically where we arbitrarily assume  $m + rq > g + \ell q$  for the purpose of sketching the situation, we have



Looking at the equation for CC\* [Eq. (130)], we see that after interchanging the order of integration and summation, we will be integrating the same complex exponential as for the AA\* and BB\* terms

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (153)$$

where the lower limit of integration will be

$$\frac{1}{\beta} \max(g + \ell q, m + r q)$$

while the upper limit will be  $T$ . Then for

**Case 1:**  $k \neq n \implies \omega_k \neq \omega_n$

$$\begin{aligned} \int_{(1/\beta) \max(g+\ell q, m+r q)}^T \xi dt &= \frac{1}{j(\omega_k - \omega_n)} e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big|_{(1/\beta) \max(g+\ell q, m+r q)}^T \\ &= \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ &\quad \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g+\ell q, m+r q) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \end{aligned} \quad (154)$$

**Case 2:**  $k = n \implies \omega_k = \omega_n$

Again, similar to the AA\* term, for  $k = n$ , the term

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (155)$$

reduces to

$$\xi = e^{j[(\omega_k/\beta)(m-g)]} \quad (156)$$

and then

$$\begin{aligned} \int_{(1/\beta) \max(g+\ell q, m+r q)}^T \xi dt &= e^{j[(\omega_k/\beta)(m-g)]} \int_{(1/\beta) \max(g+\ell q, m+r q)}^T dt \\ &= e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g + \ell q, m + r q) \right] \end{aligned} \quad (157)$$

Thus we may write the expression for  $K^2 \int_0^T CC^* dt$

$$K^2 \int_0^T CC^* dt = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-r q}^{-(r-1)q-1} \rho^{g+m} \times \varphi_3 \quad (158)$$

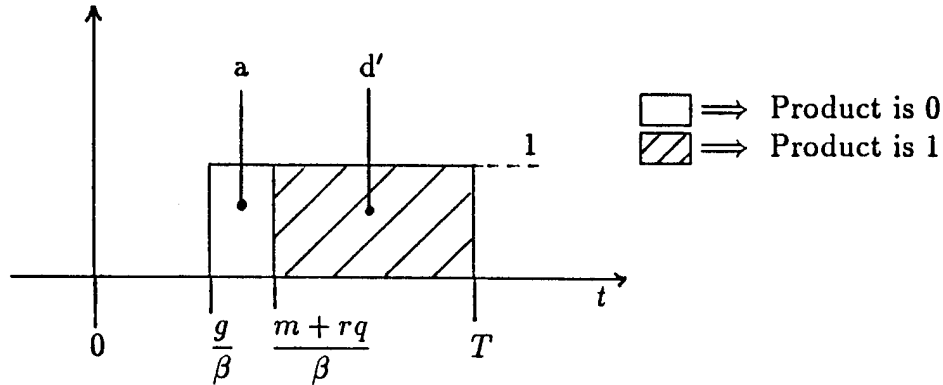
where

$$\varphi_3 = \begin{cases} b_{k,\ell} b_{n,r}^* \left[ \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g + \ell q, m + r q) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], & \text{for } k \neq n \\ b_{k,\ell} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g + \ell q, m + r q) \right], & \text{for } k = n \end{cases}$$

Now we turn to the  $2\text{Re}[\text{AC}^*]$  term [Eq. (131)]. We have the pulse multiplication of  $a d'$  where

$$a = p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) \quad d' = p_T \left( \frac{T \left( t - \left( \frac{m + r q}{\beta} \right) \right)}{\left( T - \left( \frac{m + r q}{\beta} \right) \right)} \right)$$

Looking at this graphically by arbitrarily assuming  $m + r q > g$ , we sketch the product  $a \times d'$



Then we can see that the product  $a d'$  will exist from  $(1/\beta) \max(g, m + r q)$  to  $T$ , which are the lower and upper limits of integration, respectively.

**Case 1:**  $k \neq n \Rightarrow \omega_k \neq \omega_n$

This expression has the two complex exponentials

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (159)$$

$$\varrho = e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (160)$$

to integrate. Letting

$$u = -j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]$$

and

$$du = -j(\omega_k - \omega_n)$$

the integration yields

$$\begin{aligned} \int_{(1/\beta) \max(g, m+rq)}^T \varrho dt &= \frac{1}{[-j(\omega_k - \omega_n)]} e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \Big|_{(1/\beta) \max(g, m+rq)}^T \\ &= \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ &\quad \left. - e^{-j[(\omega_k - \omega_n)(1/\beta) \max(g, m+rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \end{aligned} \quad (161)$$

and

$$\begin{aligned} \int_{(1/\beta) \max(g, m+rq)}^T \xi dt &= \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ &\quad \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g, m+rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \end{aligned} \quad (162)$$

**Case 2:**  $k = n \implies \omega_k = \omega_n$

Looking at the expression for  $2\text{Re}[AC^*]$  [Eq. (131)], we see that for  $\omega_k = \omega_n$  the two complex exponentials reduce to  $e^{j[(\omega_k/\beta)(m-g)]}$  and  $e^{-j[(\omega_k/\beta)(m-g)]}$ , which are constants to the integration. Thus, we will integrate the product of the pulse functions  $ad'$  from 0 to  $T$ . This is the only time function for this case. Then, looking at the sketch below Eq. (158)

$$\int_0^T ad' dt = \int_{(1/\beta) \max(g, m+rq)}^T dt = T - \frac{1}{\beta} \max(g, m + rq) \quad (163)$$

Thus, we are ready to write the expression for

$$K^2 \int_0^T 2\text{Re}[AC^*] dt$$



Then,

$$K^2 \int_0^T 2Re[AC^*] dt = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_4 \quad (164)$$

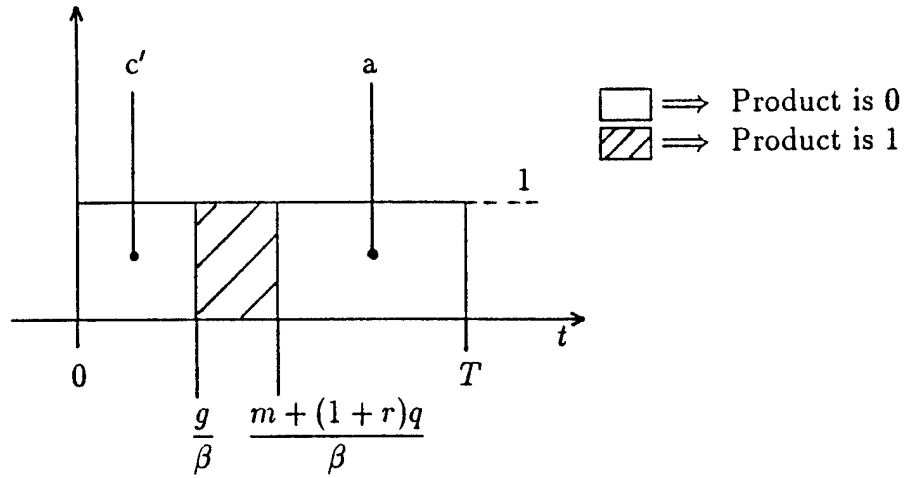
where

$$\varphi_4 = \left\{ \begin{array}{l} \rho^{g+m} b_{k,0} b_{n,r}^* \left[ \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right] \\ + \rho^{g+m} b_{k,0}^* b_{n,r} \left[ \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{-j[(\omega_k - \omega_n)(1/\beta) \max(g, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], \quad \text{for } k \neq n \\ [b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}] \rho^{g+m} \times \dots \\ \left[ T - \frac{1}{\beta} \max(g, m + rq) \right], \quad \text{for } k = n \end{array} \right.$$

We now turn to the  $2Re[AB^*]$  term [Eq. (134)]. We have the pulse multiplication  $ac'$  where

$$a = p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) \quad c' = p_T \left( \frac{\beta T(t)}{m + (1+r)q} \right)$$

To form a sketch, we arbitrarily assume  $m + (1+r)q > g$ , yielding



We see that the product will exist for  $m + (1 + r)q > g$  and will be 0 otherwise. Thus, unless  $m + (1 + r)q > g$ , the integral of  $2\text{Re}[AB^*]$  is 0. We will later create a “gating” function to account for this.

**Case 1:**  $k \neq n \implies \omega_k \neq \omega_n$

For this case where  $m + (1 + r)q > g$  we will be integrating the same complex exponentials as before

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (165)$$

and

$$\rho = e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (166)$$

We have computed these integrals before and will just substitute the new upper and lower limits in the final answer for this term

$$\text{Upper Limit : } \frac{(m + (1 + r)q)}{\beta}$$

$$\text{Lower Limit : } \frac{g}{\beta}$$

If  $m + (1 + r)q < g$ , the integrals of the two complex exponentials are zero as the product of the pulse functions  $ac' = 0$ . Again, we will create a “gating” function to allow for this.

**Case 2:**  $k = n \implies \omega_k = \omega_n$

Looking at the  $2Re[AB^*]$  term [Eq. (134)]. We see that the two complex exponentials reduce to

$$e^{j[(\omega_k/\beta)(m-g)]} \quad \text{and} \quad e^{-j[(\omega_k/\beta)(m-g)]}$$

These are constants to the integration with respect to  $t$ . Thus as before, we will only be integrating the product of the pulse functions  $ac'$  from 0 to  $T$ . The integral has value if  $m + (1+r)q > g$  and is 0 otherwise. Then, if  $m + (1+r)q > g$ , the limits of integration are the same as for the case  $\omega_k \neq \omega_n$  and

$$\int_0^T ac' dt = \int_{g/\beta}^{(m+(1+r)q)/\beta} dt = \frac{1}{\beta}[(m + (1+r)q) - g] \quad (167)$$

Now we will create the "gating" function to null the integrals if the product  $ac'$  is 0 from 0 to  $T$ .

To indicate the zeroing/nulling of all integrals if  $g \geq m + (1+r)q$ , we will create another gating function called  $G_2(g, m, r)$ .

$$G_2(g, m, r) = \begin{cases} 1, & \text{for } g < m + (1+r)q \\ 0, & \text{otherwise} \end{cases} \quad (168)$$

We are now ready to write the expression for  $K^2 \int_0^T 2Re[AB^*] dt$ . Then

$$K^2 \int_0^T 2Re[AB^*] dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_5 \quad (169)$$

where

$$\varphi_5 = \left\{ \begin{array}{ll} \left[ \left( \rho^{g+m} b_{k,0} b_{n,r}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)((m+(1+r)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \right. \\ \left. \left. \left. - e^{j[(\omega_n/\beta)(m-g)]} \right] \right) \right. \\ \left. + \left( \rho^{g+m} b_{k,0}^* b_{n,r} \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)((m+(1+r)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \right. \\ \left. \left. \left. - e^{-j[(\omega_n/\beta)(m-g)]} \right] \right) \right] G_2(g, m, r), & \text{for } k \neq n \\ \left[ b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \dots \\ \left( \frac{1}{\beta} [(m + (1+r)q) - g] \right) G_2(g, m, r), & \text{for } k = n \end{array} \right\}$$

where

$$G_2(g, m, r) = \left\{ \begin{array}{ll} 1, & \text{for } g < m + (1+r)q \\ 0, & \text{otherwise} \end{array} \right\}$$

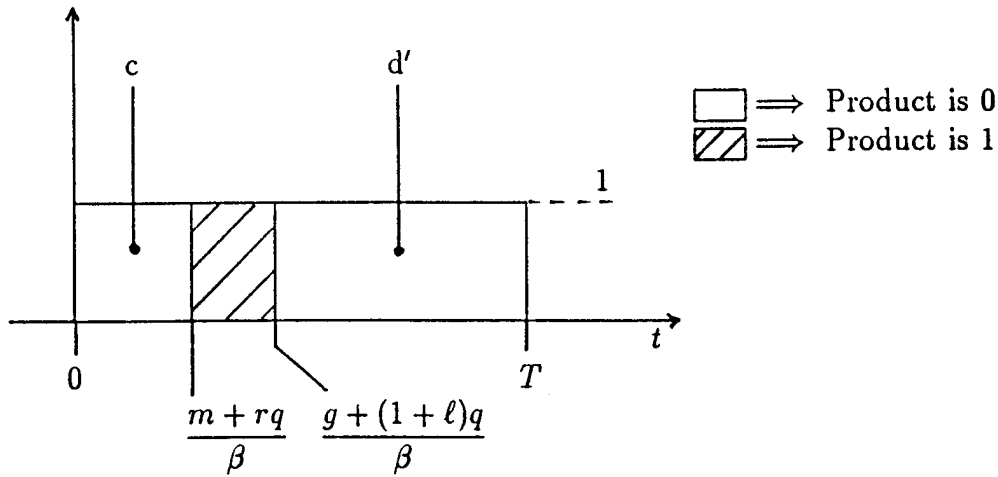
Now, finally we must write out the last (sixth) term of the ACI. We now need to find

$$K^2 \int_0^T 2Re[BC^*] dt$$

Again, we look at the expression for  $2Re[BC^*]$  [Eq. (137)] to see the pulse multiplication  $cd'$  where

$$c = p_T \left( \frac{\beta T(t)}{g + (1+\ell)q} \right) \quad d' = p_T \left( \frac{T \left( t - \left( \frac{m + rq}{\beta} \right) \right)}{\left( T - \left( \frac{m + rq}{\beta} \right) \right)} \right)$$

We sketch at this pulse multiplication by arbitrarily assuming  $g + (1+\ell)q > m + rq$ .



**Case 1:**  $k \neq n \implies \omega_k \neq \omega_n$

Here we will be integrating the complex exponentials

$$\xi = e^{j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (170)$$

and

$$\varrho = e^{-j[(\omega_k - \omega_n)t - \omega_k(g/\beta) + \omega_n(m/\beta)]} \quad (171)$$

The integrals will have the following limits

$$\text{Upper : } \frac{g + (1 + \ell)q}{\beta}$$

$$\text{Lower : } \frac{m + rq}{\beta}$$

The integrals will have value for  $m + rq < g + (1 + \ell)q$  and will be zero otherwise.

We have computed these integrals before and will just substitute the new limits in the final answer for this term. We will create another “gating” function to account for the integrals “turning off” when  $m + rq \geq g + (1 + \ell)q$

$$G_3(g, \ell, m, r) = \left\{ \begin{array}{ll} 1, & m + rq < g + (1 + \ell)q \\ 0, & \text{otherwise} \end{array} \right\} \quad (172)$$

**Case 2:**  $k = n \implies \omega_k = \omega_n$

Looking at the  $2Re [BC^*]$  term [Eq. (137)], we see again that the complex exponentials reduce to

$$e^{j[(\omega_k/\beta)(m-g)]} \quad \text{and} \quad e^{-j[(\omega_k/\beta)(m-g)]}$$

which are constants to the integration with respect to  $t$ . Thus, in a similar manner as before, we will only be integrating the product of the pulse functions  $cd'$  from 0 to  $T$ . The integral has value if  $m + rq < g + (1 + \ell)q$  and is zero otherwise. We have already created the “gating” function [Eq. (172)] to account for this turning on and off of the integral. Then, if  $m + rq < g + (1 + \ell)q$  the limits of integration are the same as for the case  $\omega_k \neq \omega_n$  and

$$\int_0^T cd' dt = \int_{(m+rq)/\beta}^{(g+(1+\ell)q)/\beta} dt = \frac{1}{\beta} [(g + (1 + \ell)q) - (m + rq)] \quad (173)$$

We are now ready to write the expression for

$$K^2 \int_0^T 2Re [BC^*] dt.$$

Then

$$K^2 \int_0^T 2Re [BC^*] dt = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_6 \quad (174)$$

where

$$\varphi_6 = \left\{ \begin{array}{ll} \left( \rho^{g+m} b_{k,\ell}^* b_{n,r}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)((g+(1+\ell)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)((m+rq)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right. \\ \left. + \rho^{g+m} b_{k,\ell}^* b_{n,r}^* \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)((g+(1+\ell)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{-j[(\omega_k - \omega_n)((m+rq)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right) G_3(g, \ell, m, r), & \text{for } k \neq n \\ \left[ b_{k,\ell} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,\ell}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \dots \\ \left( \frac{1}{\beta} [(g + (1 + \ell)q) - (m + rq)] \right) G_3(g, \ell, m, r), & \text{for } k = n \end{array} \right\}$$

where  $G_3(g, \ell, m, r)$  is defined by Eq. (172). We have now completed integrating all six terms of the ACI.

Recalling the expression for  $|s(t)|^2$  [Eq. (89)], we see that we still have to compute the following terms

$$2\text{Re}[s_B s_{ISI}^*] = 2\text{Re}[s_{ISI} s_B^*],$$

$$2\text{Re}[s_B^* s_{ACI}] = 2\text{Re}[s_B s_{ACI}^*],$$

and

$$2\text{Re}[s_{ISI}^* s_{ACI}] = 2\text{Re}[s_{ISI} s_{ACI}^*],$$

multiply each by the constant  $K^2 = P(1 - \rho)^2$  and integrate each from 0 to  $T$ . We will use either form of the above three expressions depending on the ease of computation. Recall from Eq. (66) that

$$s_B(t) = \sum_{g=0}^{q-1} \rho^g b_{0,0} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right), \quad \text{for } 0 \leq t \leq T \quad (175)$$

Let us look at  $s_B s_{ISI}^*$  term. Looking at the expression for  $s_B(t)$  during the interval  $0 \leq t \leq T$  [Eqs. (66) or (175)], we see that  $s_B(t)$  is real as  $b_{0,i} \in \{0, 1\}$ . Looking at

the expression for  $s_{ISI}(t)$  during the interval  $0 \leq t \leq T$  [Eq. (78)], we see also that  $s_{ISI}(t)$  is real as again  $b_{0,i} \in \{0, 1\}$ . Then  $s_{ISI} = s_{ISI}^*$  and  $s_B s_{ISI}^* = s_B s_{ISI}$ , and

$$2\text{Re}[s_B s_{ISI}^*] = 2s_B s_{ISI} \quad (176)$$

Performing the multiplication yields

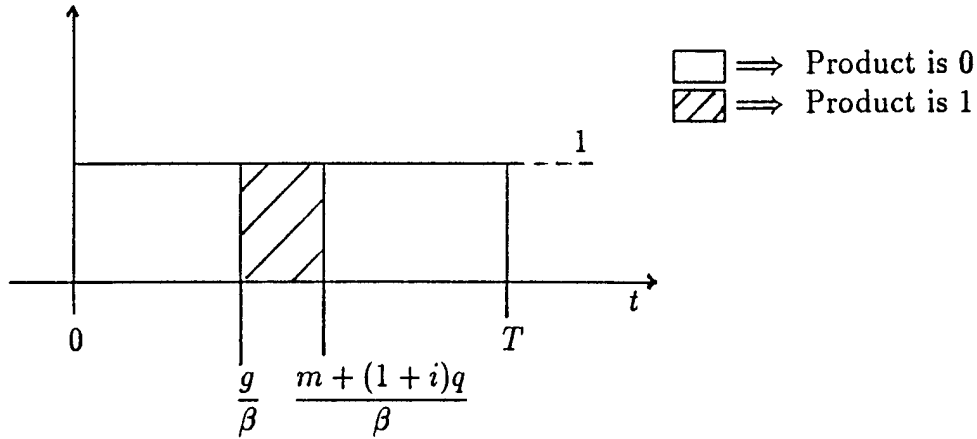
$$\begin{aligned} 2\text{Re}[s_B s_{ISI}^*] &= 2[s_B s_{ISI}] \\ &= \underbrace{2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) p_T \left( \frac{\beta T(t)}{m + (1+i)q} \right)}_e \\ &\quad + \underbrace{2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} p_T \left( \frac{T \left(t - \frac{g}{\beta}\right)}{\left(T - \frac{g}{\beta}\right)} \right) p_T \left( \frac{T \left(t - \left(\frac{m+iq}{\beta}\right)\right)}{\left(T - \left(\frac{m+iq}{\beta}\right)\right)} \right)}_f \end{aligned} \quad (177)$$

We see that when we multiply by  $K^2$  and integrate, the integral will distribute across addition to the two terms e and f. The integrals then move inside of the summations to the pulse function products. Now we need to figure the proper limits for the integration of the two pulse function products in e and f [Eq. (177)]. We note that the products of pulse functions are the only time functions in the expression  $2\text{Re}[s_B s_{ISI}^*]$ , so they are the only integrands in the expression. We examine the pulse multiplication in term e [Eq. (177)]. Let

$$E = p_T \left( \frac{T \left(t - \frac{g}{\beta}\right)}{\left(T - \frac{g}{\beta}\right)} \right) p_T \left( \frac{\beta T(t)}{m + (1+i)q} \right) \quad (178)$$

Sketching E, we arbitrarily assume  $m + (1+i)q > g$ .





The integral  $\int_0^T E dt$  will have value for  $g < m + (1+i)q$  and will be zero otherwise.

If  $g < m + (1+i)q$

$$\int_0^T E dt = \int_{g/\beta}^{(m+(1+i)q)/\beta} dt = \frac{1}{\beta} [(m + (1+i)q) - g] \quad (179)$$

Since only the pulse function product is being integrated, we will create another gating function, which also gives us the value of the above integral when it turns on.

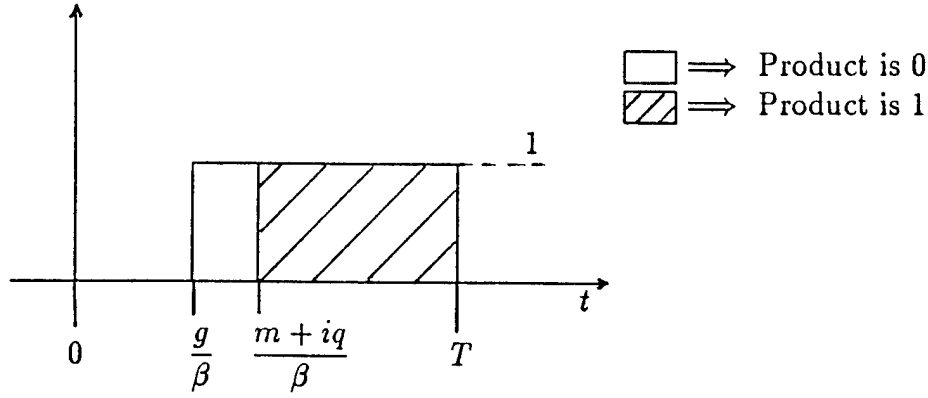
This is similar to  $G_1(g, i, m, r)$  [Eq. (112)].

$$\int_0^T E dt = G_4(g, i, m) = \begin{cases} \frac{1}{\beta} [(m + (1+i)q) - g], & g < m + (1+i)q \\ 0, & \text{otherwise} \end{cases} \quad (180)$$

Look at the pulse multiplication in term f [Eq. (177)]. Let

$$F = p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) p_T \left( \frac{T \left( t - \left( \frac{m+iq}{\beta} \right) \right)}{\left( T - \left( \frac{m+iq}{\beta} \right) \right)} \right) \quad (181)$$

Sketching F, we arbitrarily assume  $m + iq > g$ .



We see that

$$\int_0^T F dt = T - \frac{1}{\beta} \max(g, m + iq) \quad (182)$$

We can now write down the following integral

$$K^2 \int_0^T 2\text{Re}[s_B s_{IS I}^*] dt$$

Substituting integrals  $\int_0^T E dt$  and  $\int_0^T F dt$  for the pulse function products in the  $2\text{Re}[s_B s_{IS I}^*]$  expression [Eq. (177)], and multiplying by  $K^2$  yields the following result.

*Note:*

$\int_0^T E dt$  is substituted for the pulse function product in e in Eq. (177).

$\int_0^T F dt$  is substituted for the pulse function product in f in Eq. (177).

Recalling that  $K^2 = P(1 - \rho)^2$  we have

$$\begin{aligned} K^2 \int_0^T 2\text{Re}[s_B s_{IS I}^*] dt &= K^2 \int_0^T 2[s_B s_{IS I}] dt \\ &= 2P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} G_4(g, i, m) \\ &+ 2P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} \left[ T - \frac{1}{\beta} \max(g, m + iq) \right] \end{aligned} \quad (183)$$

where

$$G_4(g, i, m) = \begin{cases} \frac{1}{\beta} [(m + (1 + i)q) - g], & g < m + (1 + i)q \\ 0, & \text{otherwise} \end{cases}$$

Now we need to compute  $2Re[s_B^* s_{ACI}]$ , multiply by  $K^2$ , and integrate it from 0 to  $T$ .

We note again that  $s_B = s_B^*$  thus  $s_B^* s_{ACI} = s_B s_{ACI}$  and since  $s_{ACI}$  is complex we will distribute the  $Re[\cdot]$  operator over the summations of the  $s_B s_{ACI}$  product in the same manner as in some of the previous terms.

Computing the  $2Re[s_B^* s_{ACI}] = 2Re[s_B s_{ACI}]$  term

$$\begin{aligned} 2Re[s_B^* s_{ACI}] &= 2Re[s_B s_{ACI}] = 2Re \left[ \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{0,0} b_{k,0} e^{j\omega_k(t-(m/\beta))} \times \dots \right. \\ &\quad \underbrace{p_T \left( \frac{T}{\left(T - \frac{g}{\beta}\right)} \left(t - \frac{g}{\beta}\right) \right) p_T \left( \frac{T}{\left(T - \frac{m}{\beta}\right)} \left(t - \frac{m}{\beta}\right) \right)}_G \\ &\quad + \sum_{\substack{g=0 \\ k \neq 0}}^{q-1} \sum_{k=-M/2}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \rho^{g+m} b_{0,0} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \dots \\ &\quad \underbrace{p_T \left( \frac{T \left(t - \frac{g}{\beta}\right)}{\left(T - \frac{g}{\beta}\right)} \right) p_T \left( \frac{\beta T(t)}{m + (1 + \ell)q} \right)}_H \\ &\quad \left. + \sum_{\substack{g=0 \\ k \neq 0}}^{q-1} \sum_{k=-M/2}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \rho^{g+m} b_{0,0} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \dots \right] \end{aligned}$$

$$\underbrace{p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) p_T \left( \frac{T \left( t - \left( \frac{m + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{m + \ell q}{\beta} \right) \right)} \right)}_I \quad (184)$$

The pulse products are G, H, and I as shown above. Then we note again that  $b_{0,i}$  is real. Also,  $b_{k,\ell} \in \{0, e^{j\phi_k}\}$  and  $e^{j\omega_k(t-(m/\beta))}$  are complex. We then have

$$\begin{aligned} 2Re[s_B^* s_{ACI}] &= 2Re[s_B s_{ACI}] = \\ &\sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \left( \rho^{g+m} b_{0,0} b_{k,0} e^{j\omega_k(t-(m/\beta))} G + \rho^{g+m} b_{0,0} b_{k,0}^* e^{-j\omega_k(t-(m/\beta))} G \right) \\ &+ \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \left( \rho^{g+m} b_{0,0} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} H + \rho^{g+m} b_{0,0} b_{k,\ell}^* e^{-j\omega_k(t-(m/\beta))} H \right) \\ &+ \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \left( \rho^{g+m} b_{0,0} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} I + \rho^{g+m} b_{0,0} b_{k,\ell}^* e^{-j\omega_k(t-(m/\beta))} I \right) \end{aligned} \quad (185)$$

We see that computing  $K^2 \int_0^T 2Re[s_B^* s_{ACI}]$  will involve integrating each of the six terms inside of the three clusters of summations. The integrable part of each of these six terms is a complex exponential of the form

$$\vartheta = e^{j\omega_k(t-(m/\beta))} \quad (186)$$

or

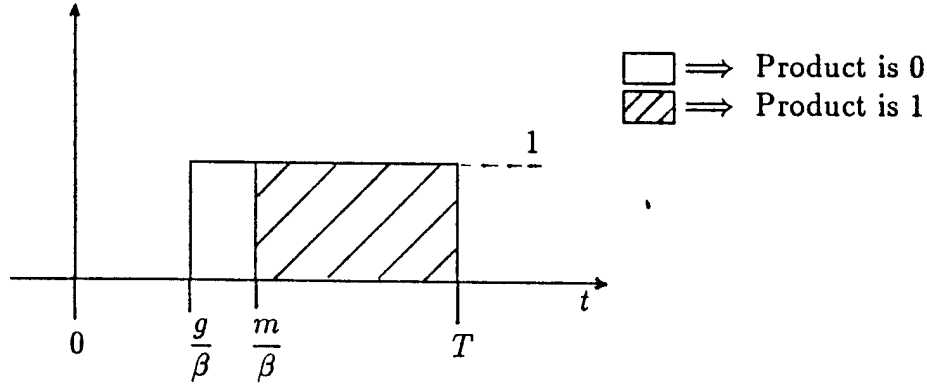
$$\mu = e^{-j\omega_k(t-(m/\beta))} \quad (187)$$

The pulse products G, H, and I will provide the limits of integration.

Looking at G [Eq. (184)]

$$G = p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) p_T \left( \frac{T \left( t - \frac{m}{\beta} \right)}{\left( T - \frac{m}{\beta} \right)} \right) \quad (188)$$

Sketching G, we arbitrarily assume  $m > g$ .



Since the height of the pulse product G is 1 from  $(1/\beta) \max(g, m)$  to T, we see that we will have the following limits of integration for the complex exponentials

$$\vartheta = e^{j\omega_k(t-(m/\beta))} \quad (189)$$

and

$$\mu = e^{-j\omega_k(t-(m/\beta))} \quad (190)$$

which are the integrands in the first cluster of summations in the  $2\text{Re}[s_B^* s_{ACI}]$  term [Eq. (185)]

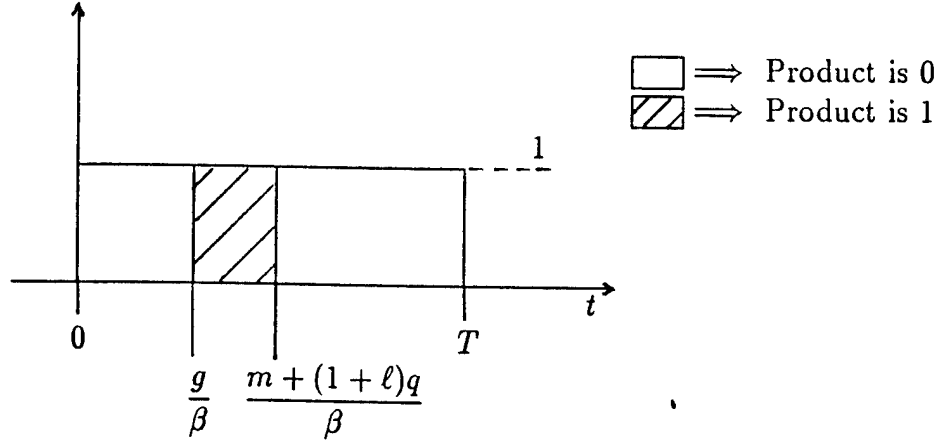
Upper Limit : T

Lower Limit :  $\frac{1}{\beta} \max(g, m)$

Turning to H [Eq. (184)]

$$H = p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) p_T \left( \frac{\beta T(t)}{m + (1 + \ell)q} \right) \quad (191)$$

Sketching H, we arbitrarily assume  $m + (1 + \ell)q > g$ .



We see that for  $g < m + (1 + \ell)q$ , the upper and lower limits of integration for the integrands  $\vartheta = e^{j\omega_k(t-(m/\beta))}$  and  $\mu = e^{-j\omega_k(t-(m/\beta))}$  contained in the second cluster of summations in the  $2\text{Re}[s_B^* s_{ACI}]$  term [Eq. (185)] will be

$$\begin{aligned} \text{Upper Limit : } & \frac{1}{\beta}(m + (1 + \ell)q) \\ \text{Lower Limit : } & \frac{g}{\beta} \end{aligned}$$

We will now create another “gating” function to null the integrals of  $\vartheta = e^{j\omega_k(t-(m/\beta))}$  and  $\mu = e^{-j\omega_k(t-(m/\beta))}$  in the second cluster of summations in the  $2\text{Re}[s_B^* s_{ACI}]$  term [Eq. (185)]. This nulling of the two integrals will occur if  $g \geq m + (1 + \ell)q$ .

So we define

$$G_5(g, \ell, m) = \begin{cases} 1, & g < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases} \quad (192)$$

Now we compute the integrals of  $\vartheta = e^{j\omega_k(t-(m/\beta))}$  and  $\mu = e^{-j\omega_k(t-(m/\beta))}$  for arbitrary upper and lower limits of integration  $\gamma_1$  and  $\alpha_1$

$$\int_{\alpha_1}^{\gamma_1} \vartheta dt = \int_{\alpha_1}^{\gamma_1} e^{j\omega_k(t-(m/\beta))} dt = \frac{1}{j\omega_k} e^{j\omega_k(t-(m/\beta))} \Big|_{\alpha_1}^{\gamma_1} \quad (193)$$

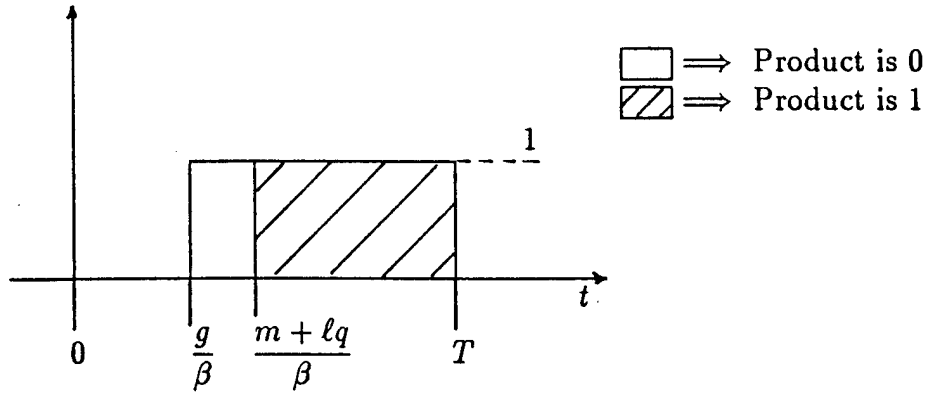
Similarly for  $\mu$

$$\int_{\alpha_1}^{\gamma_1} \mu dt = \int_{\alpha_1}^{\gamma_1} e^{-j\omega_k(t-(m/\beta))} dt = \left. \frac{1}{[-j\omega_k]} e^{-j\omega_k(t-(m/\beta))} \right|_{\alpha_1}^{\gamma_1} \quad (194)$$

Now we turn to the pulse product term I in Eq (184)

$$I = p_T \left( \frac{T \left( t - \frac{g}{\beta} \right)}{\left( T - \frac{g}{\beta} \right)} \right) p_T \left( \frac{T \left( t - \left( \frac{m + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{m + \ell q}{\beta} \right) \right)} \right) \quad (195)$$

Sketching I, we arbitrarily assume  $m + \ell q > g$ .



We see from the sketch that the upper and lower limits of integration of the integrands  $\vartheta = e^{j\omega_k(t-(m/\beta))}$  and  $\mu = e^{-j\omega_k(t-(m/\beta))}$  in the third cluster of summations in the  $2Re[s_B^* s_{ACI}]$  term [Eq. (185)] are

Upper Limit :  $T$

Lower Limit :  $\frac{1}{\beta} \max(g, m + \ell q)$

Now that we have the three sets of limits of integration for each of the clusters of summations and the indefinite integrals of  $\vartheta$  and  $\mu$ , we are prepared to write the expression for

$$K^2 \int_0^T 2Re[s_B^* s_{ACI}] dt = K^2 \int_0^T 2Re[s_B s_{ACI}] dt$$

Then, recalling that  $K^2 = P(1 - \rho)^2$

$$K^2 \int_0^T 2\text{Re}[s_B^* s_{ACI}] dt = K^2 \int_0^T 2\text{Re}[s_B s_{ACI}] dt = A_{BA} + B_{BA} + C_{BA} \quad (196)$$

where

$$A_{BA} = P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_7$$

in which

$$\begin{aligned} \varphi_7 = & \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m)-m)/\beta)} \right] \\ & + \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m)-m)/\beta)} \right] \end{aligned}$$

$$B_{BA} = P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_8$$

in which

$$\begin{aligned} \varphi_8 = & \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k((1+\ell)q/\beta)} - e^{j\omega_k((g-m)/\beta)} \right] G_5(g, \ell, m) \\ & + \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k((1+\ell)q/\beta)} - e^{-j\omega_k((g-m)/\beta)} \right] G_5(g, \ell, m) \end{aligned}$$

where

$$G_5(g, \ell, m) = \begin{cases} 1, & g < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$C_{BA} = P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_9$$



in which

$$\begin{aligned}\varphi_9 = & \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right] \\ & + \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right]\end{aligned}$$

The last term we have to compute is  $K^2 \int_0^T 2\text{Re}[s_{ISI}^* s_{ACI}] dt$ . We have already shown that  $s_{ISI}$  is real; thus

$$2\text{Re}[s_{ISI}^* s_{ACI}] = 2\text{Re}[s_{ISI} s_{ACI}] \quad (197)$$

As before, we will multiply  $s_{ISI}$  and  $s_{ACI}$ .

Look at  $s_{ISI}$  [Eq. (78)], we will let the first term be  $A$  and the second be  $B$ . Looking at  $s_{ACI}$  [Eq. (83)], we will let the first, second, and third terms be  $C$ ,  $D$ , and  $E$ , respectively. Then

$$s_{ISI} s_{ACI} = (A + B)(C + D + E) = AC + AD + AE + BC + BD + BE \quad (198)$$

Then, we compute all six of the above terms individually

$$\begin{aligned}AC = & \sum_{i=L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{0,i} b_{k,0} e^{j\omega_k(t-(m/\beta))} \times \dots \\ & p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{T \left( t - \frac{m}{\beta} \right)}{\left( T - \frac{m}{\beta} \right)} \right) \quad (199)\end{aligned}$$

$$\begin{aligned}AD = & \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \dots \\ & p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{\beta T(t)}{m + (1+\ell)q} \right) \quad (200)\end{aligned}$$

$$\begin{aligned}
AE &= \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \dots \\
&\quad p_T \left( \frac{\beta T(t)}{g + (1+i)q} \right) p_T \left( \frac{T \left( t - \left( \frac{m + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{m + \ell q}{\beta} \right) \right)} \right)
\end{aligned} \tag{201}$$

$$\begin{aligned}
BC &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} b_{0,i} b_{k,0} e^{j\omega_k(t-(m/\beta))} \times \dots \\
&\quad p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{T \left( t - \frac{m}{\beta} \right)}{\left( T - \frac{m}{\beta} \right)} \right)
\end{aligned} \tag{202}$$

$$\begin{aligned}
BD &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \dots \\
&\quad p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{\beta T(t)}{m + (1+\ell)q} \right)
\end{aligned} \tag{203}$$

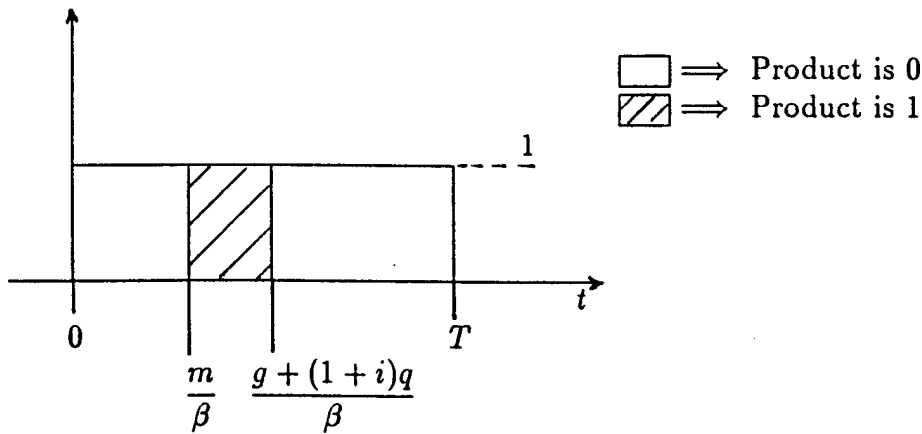
$$\begin{aligned}
BE &= \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \rho^{g+m} b_{0,i} b_{k,\ell} e^{j\omega_k(t-(m/\beta))} \times \dots \\
&\quad p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{T \left( t - \left( \frac{m + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{m + \ell q}{\beta} \right) \right)} \right)
\end{aligned} \tag{204}$$

When we distribute  $2\text{Re}[\cdot]$  across each of the terms  $AC + AD + \dots$  and use  $\text{Re}[Z] = [Z + Z^*]^2$ , the 2 will cancel with the  $1/2$ , and we will be integrating the same complex exponential terms  $\vartheta = e^{j\omega_k(t-(m/\beta))}$  and  $\mu = e^{-j\omega_k(t-(m/\beta))}$  as we did before for the  $2\text{Re}[s_B^* s_{ACI}]$  term [see Eqs. (186) and (187)]. Then, all we need to do is investigate the pulse products for each of the six terms  $AC + AD + \dots$  to figure out the proper limits of integration. Also, we may need to create other “gating” functions to null the integrals if there are specific index occurrences which make the specific pulse product 0.

The  $AC$  pulse product is

$$p_T\left(\frac{\beta T(t)}{g + (1+i)q}\right) p_T\left(\frac{T\left(t - \frac{m}{\beta}\right)}{\left(T - \frac{m}{\beta}\right)}\right)$$

Arbitrarily assuming  $g + (1+i)q > m$ , we sketch the pulse product



We see that the limits of integration will be

$$\begin{aligned} \text{Upper Limit : } & \frac{1}{\beta}(g + (1+i)q) \\ \text{Lower Limit : } & \frac{m}{\beta} \end{aligned}$$

Now we define the appropriate "gating" function

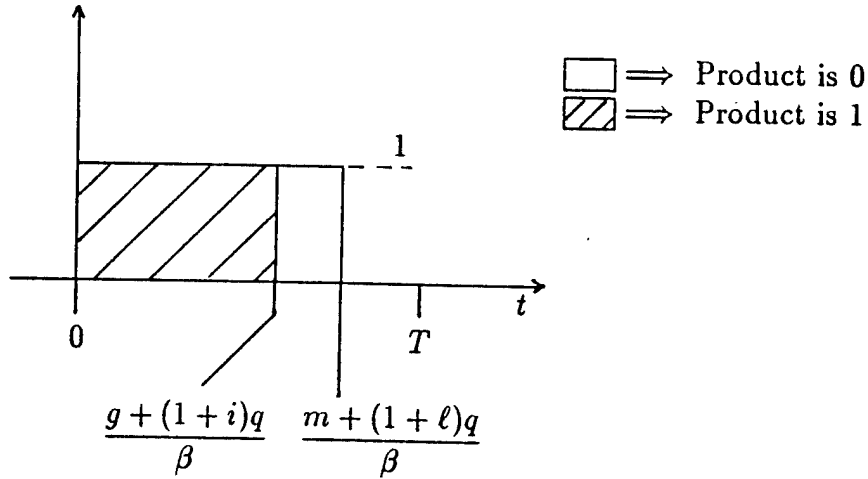
$$G_6(g, i, m) = \begin{cases} 1, & m < g + (1 + i)q \\ 0, & \text{otherwise} \end{cases} \quad (205)$$

This nulls the two applicable integrals when the pulses do not overlap.

The  $AD$  pulse product is

$$p_T \left( \frac{\beta T(t)}{g + (1 + i)q} \right) p_T \left( \frac{\beta T(t)}{m + (1 + \ell)q} \right)$$

Arbitrarily assuming  $m + (1 + \ell)q > g + (1 + i)q$ , we sketch the pulse product.



We see that the limits of integration will be

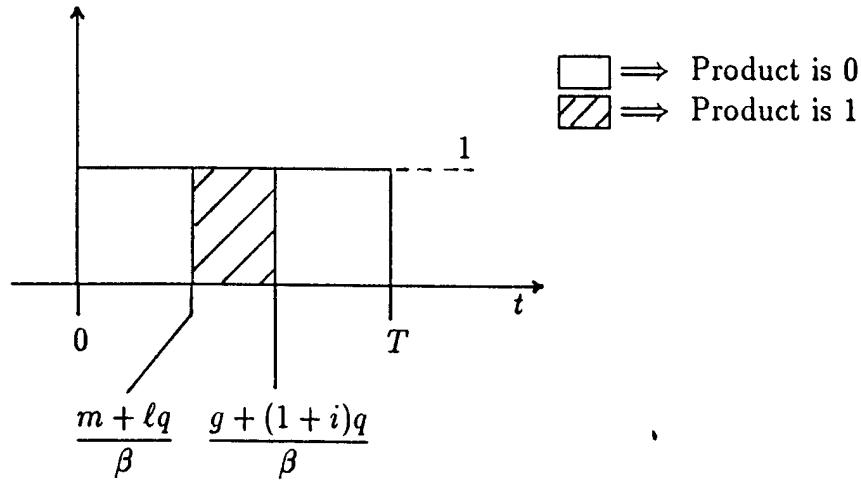
$$\text{Upper Limit : } \frac{1}{\beta} \min(g + (1 + i)q, m + (1 + \ell)q)$$

$$\text{Lower Limit : } 0$$

The  $AE$  pulse product is

$$p_T \left( \frac{\beta T(t)}{g + (1 + i)q} \right) p_T \left( \frac{T \left( t - \left( \frac{m + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{m + \ell q}{\beta} \right) \right)} \right)$$

Arbitrarily assuming  $g + (1 + i)q > m + \ell q$ , we sketch the pulse product.



We see that the limits of integration will be

$$\text{Upper Limit : } \frac{1}{\beta}(g + (1 + i)q)$$

$$\text{Lower Limit : } \frac{1}{\beta}(m + \ell q)$$

Now we define an appropriate gating function

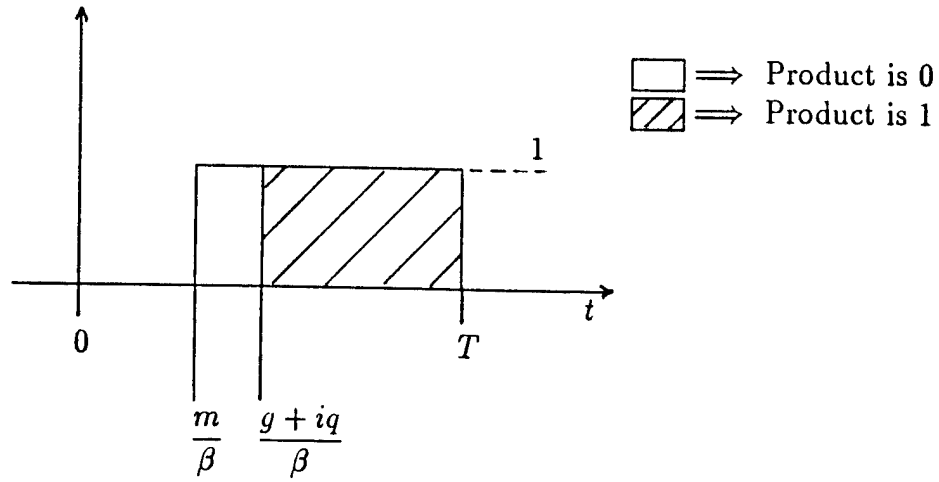
$$G_7(g, i, \ell, m) = \begin{cases} 1, & m + \ell q < g + (1 + i)q \\ 0, & \text{otherwise} \end{cases} \quad (206)$$

This nulls the two applicable integrals when the pulses do not overlap.

The  $BC$  pulse product is

$$p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{T \left( t - \frac{m}{\beta} \right)}{\left( T - \frac{m}{\beta} \right)} \right)$$

Arbitrarily assuming  $g + iq > m$ , we sketch the pulse product.



We see that the limits of integration will be

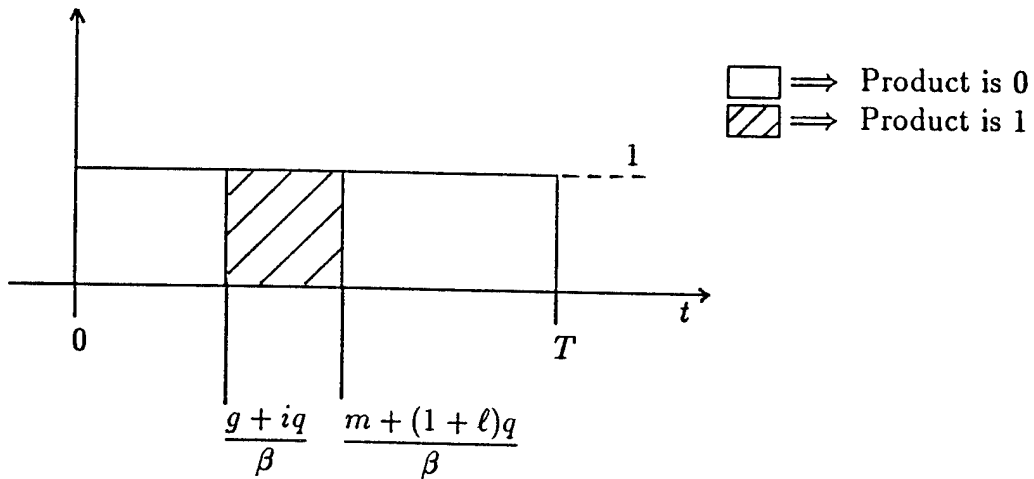
$$\text{Upper Limit : } T$$

$$\text{Lower Limit : } \frac{1}{\beta} \max(g + iq, m)$$

The *BD* pulse product is

$$p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{\beta T(t)}{m + (1 + \ell)q} \right)$$

Arbitrarily assuming  $g + iq < m + (1 + \ell)q$ , we sketch the pulse product.



We see that the limits of integration will be

$$\text{Upper Limit : } \frac{1}{\beta}(m + (1 + \ell)q)$$

$$\text{Lower Limit : } \frac{1}{\beta}(g + iq)$$

We now define an appropriate gating function

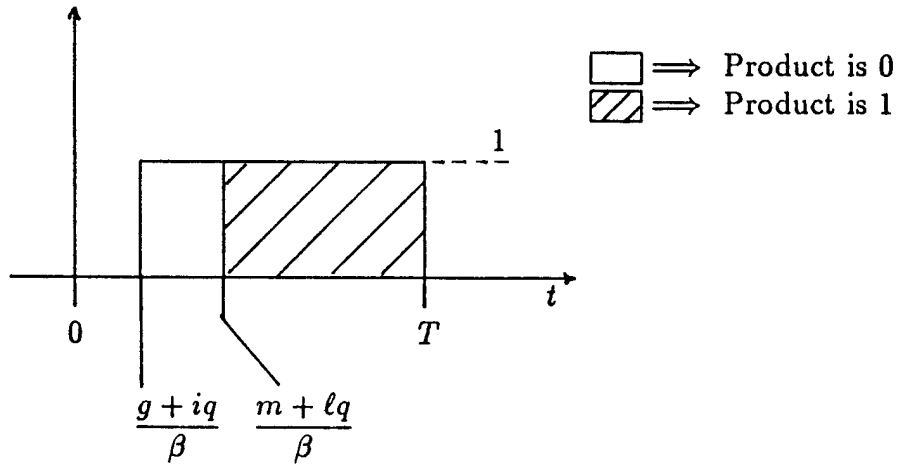
$$G_8(g, i, \ell, m) = \left\{ \begin{array}{ll} 1, & g + iq < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{array} \right\} \quad (207)$$

This nulls the two applicable integrals when the pulses do not overlap.

The *BE* pulse product is

$$p_T \left( \frac{T \left( t - \left( \frac{g + iq}{\beta} \right) \right)}{\left( T - \left( \frac{g + iq}{\beta} \right) \right)} \right) p_T \left( \frac{T \left( t - \left( \frac{m + \ell q}{\beta} \right) \right)}{\left( T - \left( \frac{m + \ell q}{\beta} \right) \right)} \right)$$

Arbitrarily assuming  $m + \ell q > g + iq$ , we sketch the pulse product.



We see that the limits of integration will be

$$\text{Upper Limit : } T$$

$$\text{Lower Limit : } \frac{1}{\beta} \max(g + iq, m + \ell q)$$

We will now apply the methods outlined below Eq. (204) and perform the  $2\text{Re}[\cdot]$  operation on each of the six terms of  $s_{ISI}^* s_{ACI} = s_{ISI} s_{ACI}$ . We will then integrate the resulting  $\vartheta = e^{j\omega_k(t-(m/\beta))}$  and  $\mu = e^{-j\omega_k(t-(m/\beta))}$  terms in each of the six clusters of summations and multiply by the proper gating function, if necessary, to compute

$$K^2 \int_0^T 2\text{Re}[s_{ISI}^* s_{ACI}] dt = K^2 \int_0^T 2\text{Re}[s_{ISI} s_{ACI}] dt$$

Recalling that  $K^2 = P(1 - \rho)^2$

$$K^2 \int_0^T 2\text{Re}[s_{ISI}^* s_{ACI}] dt = K^2 \int_0^T 2\text{Re}[s_{ISI} s_{ACI}] dt = AC_R + AD_R + AE_R + BC_R + BD_R + BE_R$$

where

$$AC_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{10} \quad (208)$$

in which

$$\begin{aligned} \varphi_{10} = & \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_* \right] G_6(g, i, m) \\ & + \rho^{g+m} b_{0,i} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_* \right] G_6(g, i, m) \end{aligned}$$

where

$$G_6(g, i, m) = \begin{cases} 1, & m < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

\* Note:  $e^{\pm j\omega_k(m/\beta-m/\beta)} = e^{j0} = 1$

$$AD_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{11} \quad (209)$$

in which

$$\varphi_{11} = \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta) \min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{j\omega_k[-m/\beta]} \right]$$



$$\begin{aligned}
& + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta) \min(g+(1+i)q, m+(1+\ell)q) - m/\beta]} - e^{-j\omega_k[-m/\beta]} \right] \\
AE_R = & P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{12} \quad (210)
\end{aligned}$$

in which

$$\begin{aligned}
\varphi_{12} = & \left[ \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(g+(1+i)q) - m/\beta]} - e^{j\omega_k[\ell q/\beta]} \right] \right. \\
& \left. + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(g+(1+i)q) - m/\beta]} - e^{-j\omega_k[\ell q/\beta]} \right] \right] G_7(g, i, \ell, m)
\end{aligned}$$

where

$$G_7(g, i, \ell, m) = \begin{cases} 1, & m + \ell q < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

$$BC_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{13} \quad (211)$$

in which

$$\begin{aligned}
\varphi_{13} = & \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[T - (m/\beta)]} - e^{j\omega_k[(1/\beta) \max(g+iq, m) - m/\beta]} \right] \\
& + \rho^{g+m} b_{0,i} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[T - (m/\beta)]} - e^{-j\omega_k[(1/\beta) \max(g+iq, m) - m/\beta]} \right]
\end{aligned}$$

$$BD_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{14} \quad (212)$$

in which

$$\begin{aligned}
\varphi_{14} = & \left[ \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(1+\ell)q]} - e^{j\omega_k[(1/\beta)(g+iq) - m/\beta]} \right] \right. \\
& \left. + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(1+\ell)q]} - e^{-j\omega_k[(1/\beta)(g+iq) - m/\beta]} \right] \right] G_8(g, i, \ell, m)
\end{aligned}$$

where

$$G_8(g, i, \ell, m) = \begin{cases} 1, & g + iq < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$BE_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{15} \quad (213)$$

in which

$$\begin{aligned} \varphi_{15} = & \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k [T - (m/\beta)]} - e^{j\omega_k [(1/\beta) \max(g+iq, m+\ell q) - m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k [T - (m/\beta)]} - e^{-j\omega_k [(1/\beta) \max(g+iq, m+\ell q) - m/\beta]} \right] \end{aligned}$$

We have now computed all terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt \quad [\text{see Eq. (89)}] \quad (214)$$

Now we recall from Eq. (89) that

$$\begin{aligned} \int_0^T |s(t)|^2 dt &= K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt \\ &= K^2 \int_0^T |s_B|^2 dt + K^2 \int_0^T |s_{ISI}|^2 dt + K^2 \int_0^T |s_{ACI}|^2 dt \\ &+ K^2 \int_0^T 2\text{Re}[s_B s_{ISI}^*] dt + K^2 \int_0^T 2\text{Re}[s_B^* s_{ACI}] dt + K^2 \int_0^T [s_{ISI}^* s_{ACI}] dt \end{aligned} \quad (215)$$

We will now extract the answers for the above six integrals from the previous rather lengthy derivation so that we will have the final answer for

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T |s'(t)|^2 dt \quad (216)$$

in one single place. Now, before we consolidate the terms we once again restate

$\max(x_1, x_2):$	Choose largest of $x_1$ or $x_2$ , which are both positive. If $x_1 = x_2$ , then $\max(x_1, x_2) = x_1 = x_2$ .
$\min(x_1, x_2):$	Choose smallest of $x_1$ or $x_2$ , which are both positive. If $x_1 = x_2$ , then $\min(x_1, x_2) = x_1 = x_2$ .

Also recall that  $K = \sqrt{P}(1 - \rho)$  and  $K^2 = P(1 - \rho)^2$ . Then we have

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt \quad (217)$$

which has the following six integrals given by Eq. (215)

$$K^2 \int_0^T |s_B|^2 dt = P(1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left( T - \left( \frac{\max(g, m)}{\beta} \right) \right) \quad (218)$$

For

$$K^2 \int_0^T |s_{ISI}|^2 dt$$

we recall the result of Eq. (117)

$$K^2 \int_0^T s_{ISI} s_{ISI}^* dt = K^2 \int_0^T |s_{ISI}|^2 dt = K^2 \left[ \int_0^T \mathcal{A} dt + \int_0^T \mathcal{B} dt \int_0^T \mathcal{C} dt + \int_0^T \mathcal{D} dt \right] \quad (219)$$

$$K^2 \int_0^T \mathcal{A} dt = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_0$$

in which

$$\begin{aligned} \varphi_0 &= \rho^{g+m} b_{0,i} b_{0,r} \left( \frac{\min(g + (1+i)q, m + (1+r)q)}{\beta} \right) \\ K^2 \int_0^T \mathcal{B} dt + K^2 \int_0^T \mathcal{C} dt &= 2K^2 \int_0^T \mathcal{B} dt = \\ 2P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} G_1(g, i, m, r) \end{aligned}$$

where

$$G_1(g, i, m, r) = \begin{cases} \frac{1}{\beta} [(g + (1+i)q) - (m + rq)], & \text{for } m + rq < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

and

$$\begin{aligned} K^2 \int_0^T \mathcal{D} dt &= P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{r=-L_0}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} b_{0,i} b_{0,r} \\ &\quad \left[ T - \left( \frac{\max(g + iq, m + rq)}{\beta} \right) \right] \end{aligned}$$

We know from Eq. (138) that

$$\begin{aligned}
K^2 \int_0^T |s_{ACI}|^2 dt &= K^2 \int_0^T AA^* dt + K^2 \int_0^T BB^* dt + K^2 \int_0^T CC^* dt \\
&\quad + K^2 \int_0^T 2\text{Re}[AC^*] dt + K^2 \int_0^T 2\text{Re}[AB^*] dt \\
&\quad + K^2 \int_0^T 2\text{Re}[BC^*] dt
\end{aligned} \tag{220}$$

So we have the following six ACI components

$$K^2 \int_0^T AA^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \rho^{g+m} \times \varphi_1 \tag{221}$$

where

$$\varphi_1 = \left\{ \begin{array}{ll} b_{k,0} b_{n,0}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \\ \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g,m) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right], & \text{for } k \neq n \\ \underbrace{b_{k,0} b_{k,0}^*}_{|b_{k,0}|^2} e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g, m) \right], & \text{for } k = n \end{array} \right\}$$

$$K^2 \int_0^T BB^* dt = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \rho^{g+m} \times \varphi_2 \tag{222}$$

where

$$\varphi_2 = \left\{ \begin{array}{ll} b_{k,\ell} b_{n,r}^* \left[ \frac{e^{j[-\omega_k(g/\beta) + \omega_n(m/\beta)]}}{j(\omega_k - \omega_n)} \left( e^{j[(\omega_k - \omega_n)(1/\beta) \min(g+(1+\ell)q, m+(1+r)q)]} - 1 \right) \right], & \text{for } k \neq n \\ b_{k,\ell} b_{k,r}^* \left( e^{j[(\omega_k/\beta)(m-g)]} \left[ \frac{1}{\beta} \min(g + (1+\ell)q, m + (1+r)q) \right] \right), & \text{for } k = n \end{array} \right\}$$

$$K^2 \int_0^T CC^* dt = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-\ell q}^{-(\ell-1)q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \rho^{g+m} \times \varphi_3 \quad (223)$$

where

$$\varphi_3 = \left\{ \begin{array}{l} b_{k,\ell} b_{n,r}^* \left[ \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g + \ell q, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], \quad \text{for } k \neq n \\ b_{k,\ell} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} \left[ T - \frac{1}{\beta} \max(g + \ell q, m + rq) \right], \quad \text{for } k = n \end{array} \right\}$$

$$K^2 \int_0^T 2\text{Re}[AC^*] = P(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_4 \quad (224)$$

where

$$\varphi_4 = \left\{ \begin{array}{l} \rho^{g+m} b_{k,0} b_{n,r}^* \left[ \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{j[(\omega_k - \omega_n)(1/\beta) \max(g, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right] \\ + \rho^{g+m} b_{k,0}^* b_{n,r} \left[ \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)T - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ \left. \left. - e^{-j[(\omega_k - \omega_n)(1/\beta) \max(g, m + rq) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right], \quad \text{for } k \neq n \\ [b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]}] \rho^{g+m} \times \dots \\ \left[ T - \frac{1}{\beta} \max(g, m + rq) \right], \quad \text{for } k = n \end{array} \right\}$$

$$K^2 \int_0^T 2\text{Re}[AB^*] = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{q-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-(1+r)q+1}^{(-rq)-1} \varphi_5 \quad (225)$$

where

$$\varphi_5 = \left\{ \begin{array}{ll} \left[ \left( \rho^{g+m} b_{k,0}^* b_{n,r}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)((m+(1+r)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \right. \\ \left. \left. \left. - e^{j[(\omega_n/\beta)(m-g)]} \right] \right) \right. \\ \left. + \left( \rho^{g+m} b_{k,0}^* b_{n,r}^* \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)((m+(1+r)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \right. \\ \left. \left. \left. - e^{-j[(\omega_n/\beta)(m-g)]} \right] \right) \right] G_2(g, m, r), & \text{for } k \neq n \\ \left[ b_{k,0} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,0}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \dots & \\ \left( \frac{1}{\beta} [(m + (1+r)q) - g] \right) G_2(g, m, r), & \text{for } k = n \end{array} \right\}$$

where

$$G_2(g, m, r) = \begin{cases} 1, & \text{for } g < m + (1+r)q \\ 0, & \text{otherwise} \end{cases}$$

$$K^2 \int_0^T 2\text{Re}[BC^*] = P(1-\rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{g=-(1+\ell)q+1}^{(-\ell q)-1} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} \sum_{r=-L}^{-1} \sum_{m=-rq}^{-(r-1)q-1} \varphi_6 \quad (226)$$

where

$$\varphi_6 = \left\{ \begin{aligned} & \left( \rho^{g+m} b_{k,\ell}^* b_{n,r}^* \frac{1}{j(\omega_k - \omega_n)} \left[ e^{j[(\omega_k - \omega_n)((g+(1+\ell)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ & \quad \left. \left. - e^{j[(\omega_k - \omega_n)((m+rq)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right. \\ & \quad \left. + \rho^{g+m} b_{k,\ell}^* b_{n,r} \frac{1}{[-j(\omega_k - \omega_n)]} \left[ e^{-j[(\omega_k - \omega_n)((g+(1+\ell)q)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right. \right. \\ & \quad \left. \left. - e^{-j[(\omega_k - \omega_n)((m+rq)/\beta) - \omega_k(g/\beta) + \omega_n(m/\beta)]} \right] \right) G_3(g, \ell, m, r), \quad \text{for } k \neq n \\ & \left[ b_{k,\ell} b_{k,r}^* e^{j[(\omega_k/\beta)(m-g)]} + b_{k,\ell}^* b_{k,r} e^{-j[(\omega_k/\beta)(m-g)]} \right] \rho^{g+m} \times \dots \\ & \left( \frac{1}{\beta} [(g + (1 + \ell)q) - (m + rq)] \right) G_3(g, \ell, m, r), \quad \text{for } k = n \end{aligned} \right\}$$

where  $G_3(g, \ell, m, r)$  is defined in Eq. (172).

Now, for the last three terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt$$

which are

$$K^2 \int_0^T 2\text{Re}[s_B s_{ISI}^*] dt; \quad K^2 \int_0^T 2\text{Re}[s_B^* s_{ACI}] dt$$

and

$$K^2 \int_0^T 2\text{Re}[s_{ISI}^* s_{ACI}] dt$$

$$K^2 \int_0^T 2\text{Re}[s_B s_{ISI}^*] dt = K^2 \int_0^T 2[s_B s_{ISI}] dt =$$

$$\begin{aligned} & 2P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-(1+i)q+1}^{(-iq)-1} \rho^{g+m} b_{0,0} b_{0,i} G_4(g, i, m) \\ & + 2P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{i=-L_0}^{-1} \sum_{m=-iq}^{-(i-1)q-1} \rho^{g+m} b_{0,0} b_{0,i} \left[ T - \frac{1}{\beta} \max(g, m + iq) \right] \end{aligned} \quad (227)$$

where

$$G_4(g, i, m) = \begin{cases} \frac{1}{\beta}[(m + (1 + i)q) - g], & g < m + (1 + i)q \\ 0, & \text{otherwise} \end{cases}$$

$$K^2 \int_0^T 2\text{Re}[s_B^* s_{ACI}] dt = K^2 \int_0^T 2\text{Re}[s_B s_{ACI}] dt = A_{BA} + B_{BA} + C_{BA}$$

$$A_{BA} = P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_7$$

in which

$$\begin{aligned} \varphi_7 = & \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m)-m)/\beta)} \right] \\ & + \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m)-m)/\beta)} \right] \end{aligned}$$

$$B_{BA} = P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_8$$

where

$$\begin{aligned} \varphi_8 = & \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k((1+\ell)q/\beta)} - e^{j\omega_k((g-m)/\beta)} \right] G_5(g, \ell, m) \\ & + \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k((1+\ell)q/\beta)} - e^{-j\omega_k((g-m)/\beta)} \right] G_5(g, \ell, m) \end{aligned}$$

where

$$G_5(g, \ell, m) = \begin{cases} 1, & g < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$C_{BA} = P(1 - \rho)^2 \sum_{g=0}^{q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_9$$



$$\begin{aligned}\varphi_9 &= \rho^{g+m} b_{0,0} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right] \\ &\quad + \rho^{g+m} b_{0,0} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m+\ell q)-m)/\beta)} \right] \quad (228)\end{aligned}$$

$$\begin{aligned}K^2 \int_0^T 2\text{Re}[s_{ISI}^* s_{ACI}] dt &= K^2 \int_0^T 2\text{Re}[s_{ISI} s_{ACI}] dt \\ &= AC_R + AD_R AE_R + BC_R + BD_R + BE_R\end{aligned}$$

$$AC_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{10} \quad (229)$$

where

$$\begin{aligned}\varphi_{10} &= \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_* \right] G_6(g, i, m) \\ &\quad + \rho^{g+m} b_{0,i} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - \underbrace{1}_* \right] G_6(g, i, m)\end{aligned}$$

where

$$G_6(g, i, m) = \begin{cases} 1, & m < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

\* Note:  $e^{\pm j\omega_k(m/\beta-m/\beta)} = e^{j0} = 1$

$$AD_R = P(1-\rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{11} \quad (230)$$

in which

$$\begin{aligned}\varphi_{11} &= \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta) \min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{j\omega_k[-m/\beta]} \right] \\ &\quad + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta) \min(g+(1+i)q, m+(1+\ell)q)-m/\beta]} - e^{-j\omega_k[-m/\beta]} \right]\end{aligned}$$

$$AE_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-(1+i)q+1}^{(-iq)-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{12} \quad (231)$$

in which

$$\begin{aligned} \varphi_{12} = & \left[ \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{j\omega_k[\ell q/\beta]} \right] \right. \\ & \left. + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(g+(1+i)q)-m/\beta]} - e^{-j\omega_k[\ell q/\beta]} \right] \right] G_7(g, i, \ell, m) \end{aligned}$$

where

$$G_7(g, i, \ell, m) = \begin{cases} 1, & m + \ell q < g + (1+i)q \\ 0, & \text{otherwise} \end{cases}$$

$$BC_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{m=0}^{q-1} \varphi_{13} \quad (232)$$

in which

$$\begin{aligned} \varphi_{13} = & \rho^{g+m} b_{0,i} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[T-(m/\beta)]} - e^{j\omega_k[(1/\beta)\max(g+iq,m)-m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[(1/\beta)\max(g+iq,m)-m/\beta]} \right] \end{aligned}$$

$$BD_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-(1+\ell)q+1}^{(-\ell q)-1} \varphi_{14} \quad (233)$$

in which

$$\begin{aligned} \varphi_{14} = & \left[ \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(1/\beta)(1+\ell)q]} - e^{j\omega_k[(1/\beta)(g+iq)-m/\beta]} \right] \right. \\ & \left. + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(1/\beta)(1+\ell)q]} - e^{-j\omega_k[(1/\beta)(g+iq)-m/\beta]} \right] \right] G_8(g, i, \ell, m) \end{aligned}$$

where

$$G_8(g, i, \ell, m) = \begin{cases} 1, & g + iq < m + (1 + \ell)q \\ 0, & \text{otherwise} \end{cases}$$

$$BE_R = P(1 - \rho)^2 \sum_{i=-L_0}^{-1} \sum_{g=-iq}^{-(i-1)q-1} \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\ell=-L}^{-1} \sum_{m=-\ell q}^{-(\ell-1)q-1} \varphi_{15} \quad (234)$$

in which

$$\begin{aligned} \varphi_{15} = & \rho^{g+m} b_{0,i} b_{k,\ell} \frac{1}{j\omega_k} \left[ e^{j\omega_k [T - (m/\beta)]} - e^{j\omega_k [(1/\beta) \max(g+iq, m+\ell q) - m/\beta]} \right] \\ & + \rho^{g+m} b_{0,i} b_{k,\ell}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k [T - (m/\beta)]} - e^{-j\omega_k [(1/\beta) \max(g+iq, m+\ell q) - m/\beta]} \right] \end{aligned}$$

We have now laid down all terms of

$$\int_0^T |s(t)|^2 dt = K^2 \int_0^T s'(t) s'(t)^* dt = K^2 \int_0^T |s'(t)|^2 dt$$

This will be used to compute the detection statistic  $X$  where

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt$$

where  $\mathcal{R}$  is the responsivity of the photodetector (A/W). This will be used to compute probabilities of bit error for the dense WDM system.



## APPENDIX B

### LIMITED CASE OF THE COMPLETE MODEL (10 PROGRAMMABLE TERMS) AND RESULTING PROBABILITY OF BIT ERROR EQUATIONS

In Appendix A we derived the complete expression for  $\int_0^T |s(t)|^2 dt$ . Referring to the OOK receiver structure in Fig. 1, we note that the deterministic signal component of the random variable appearing at the output of the integrator is

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \quad (235)$$

where  $\mathcal{R}$  is the responsivity of the photodetector (A/W) and  $s(t)$  is the complex baseband output of the Fabry-Perot filter. Note that the photodetector detects  $s(t)$  and produces an output current  $\mathcal{R}|s(t)|^2$ . We will call the integrator output the decision variable  $Y$  where

$$Y = X + N \quad (236)$$

We note that  $N$  is a Gaussian random variable with mean zero and variance  $N_0 T$

$$N \sim \mathbf{N}(0, N_0 T) \quad (237a)$$

and

$$N = \int_0^T n(t) dt \quad (237b)$$

where  $n(t)$  is the postdetection additive white Gaussian noise and  $N_0$  ( $\text{A}^2/\text{Hz}$ ) is the two-sided current spectral density of  $n(t)$ .

Now we will set up a fairly limited but realistic case for which we will program the appropriate terms derived in Appendix A and generate the probability of bit error. Letting

$$\begin{aligned} L_0 &= 1 & L &= 0 \\ \phi_k &= 0 & \omega_k &= \frac{2\pi k I}{T} \end{aligned}$$

$\omega_k = 2\pi kI/T$  is a special case of the radian frequency spacing between Channel 0 and Channel  $k$ . Recall again that  $I$  is the normalized channel spacing integer ( $I > 0$ ), and  $T$  is the data bit period (s).  $L = 0$  and  $\phi_k = 0$  means that we will only model and deal with adjacent channel interference (ACI) from the bits in the adjacent channels which occur during the detection window  $0 \leq t \leq T$ . Recall that the adjacent channels are symmetric in frequency around Channel 0, the channel of interest.  $L_0 = 1$  means that we will model the effects of a single ISI bit trailing the detected bit of interest  $b_{0,0}$  in Channel 0.  $\phi_k = 0$  means that the phase offset between Channel 0 and Channel  $k$  is 0, or that all adjacent channels are bit synchronous with Channel 0.

Then, since  $\phi_k = 0$ ,  $L = 0$ , and  $b_{k,\ell} \in \{0, e^{j\phi_k}\}$ , we let

$$b_{-M/2,0} = b_{-M/2+1,0} = \cdots = b_{-1,0} = b^- \quad \text{and} \quad b^- \in \{0, 1\} \quad (238)$$

$b^-$  is the left adjacent channel bit pattern and all  $M/2$  of the left channel bits will be 1 or 0 simultaneously during detection window  $0 \leq t \leq T$ . And again, since  $\phi_k = 0$ ,  $L = 0$ , and  $b_{k,\ell} \in \{0, e^{j\phi_k}\}$ , we let

$$b_{M/2,0} = b_{M/2-1,0} = \cdots = b_{1,0} = b^+ \quad \text{and} \quad b^+ \in \{0, 1\} \quad (239)$$

$b^+$  is the right adjacent channel bit pattern and all  $M/2$  of the right channel bits will be 1 or 0 simultaneously during the detection window  $0 \leq t \leq T$ . Also, we know  $L_0 = 1$ , and  $b_{0,i} \in \{0, 1\}$ , so using Eq. (1) or Eq. (41) we have

$$b_{0,-1} \in \{0, 1\} \quad \text{and} \quad b_{0,0} \in \{0, 1\} \quad (240)$$

We define an ACI/ISI bit pattern

$$\psi_p = \{b^-, b^+, b_{0,-1}\} \quad (241)$$

Note that  $b^-$ ,  $b^+$ , and  $b_{0,-1}$  are 0 or 1 with probability 1/2 yielding eight possible values of  $\psi_p$ .

Now let use define an ACI/ISI bit pattern set  $\psi$

$$\psi = \{\psi_1, \psi_2, \dots, \psi_8\} \quad (242)$$

We can denote each individual element in the set  $\psi$  as  $\psi_p$  where  $p = 1, 2, 3, \dots, 8$  as there are eight possible values of  $\psi_p$ , and

$$\psi = \{\psi_p\} \quad (243)$$

where  $p = 1, \dots, 8$ . Now we define

$$X_0(\psi_p) = X(\psi_p, b_{0,0} = 0) \quad \text{or} \quad X \text{ evaluated at the current value of } \psi_p \text{ with } b_{0,0} = 0 \quad (244)$$

$$X_1(\psi_p) = X(\psi_p, b_{0,0} = 1) \quad \text{or} \quad X \text{ evaluated at the current value of } \psi_p \text{ with } b_{0,0} = 1 \quad (245)$$

It can be shown that the conditional probability of error for the dense WDM system given the ACI/ISI bit pattern  $\psi_p$  is [2, 4]

$$P(\text{error}|\psi_p) = \frac{1}{2}Q\left(\frac{X_1(\psi_p) - V_T}{\sqrt{N_0T}}\right) + \frac{1}{2}Q\left(\frac{V_T - X_0(\psi_p)}{\sqrt{N_0T}}\right) \quad (246)$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \quad (247)$$

There are eight possible members of the set  $\psi$

$$\psi = \left\{ \begin{array}{ll} \psi_1 = 0 & 0 & 0 & \psi_5 = 1 & 0 & 0 \\ \psi_2 = 0 & 0 & 1 & \psi_6 = 1 & 0 & 1 \\ \psi_3 = 0 & 1 & 0 & \psi_7 = 1 & 1 & 0 \\ \psi_4 = 0 & 1 & 1 & \psi_8 = 1 & 1 & 1 \end{array} \right\} \quad (248)$$

$V_T$  is the detection threshold where

$$V_T = \frac{X_0 \max + X_1 \min}{2} \quad (249)$$

and

$$X_0 \max = \max_{\{\psi_p\}} (X_0(\psi_p)) \quad (250)$$

i.e., to find  $X_0 \max$ , compute all eight values of  $X_0$  [see Eq. (244)] and then choose the maximum, and

$$X_1 \min = \min_{\{\psi_p\}} (X_1(\psi_p)) \quad (251)$$

i.e., to find  $X_1 \min$ , compute all eight values of  $X_1$  [see Eq. (245)] and then choose the minimum. Then, by the law of total probability, the probability of bit error is

$$P_b = P(\text{error}|\psi_1)P(\psi_1) + P(\text{error}|\psi_2)P(\psi_2) + \cdots + P(\text{error}|\psi_8)P(\psi_8) \quad (252)$$

Assuming all bit patterns are equiprobable

$$P(\psi_1) = P(\psi_2) = \cdots = P(\psi_8) = \frac{1}{8} \quad (253)$$

Then we can easily see that

$$P_b = \frac{1}{8} \sum_{p=1}^8 P(\text{error}|\psi_p) \quad (254)$$

Again, we note that

$$X = \mathcal{R} \int_0^T |s(t)|^2 dt \quad (255)$$

where  $\int_0^T |s(t)|^2 dt$  was computed in Appendix A and consisted of 21 terms (“clusters of summations”) [see Eqs. (218)–(234)]. Given our parameters  $L_0 = 1$ ,  $L = 0$ ,  $\phi_k = 0$ , and  $\omega_k = 2\pi kI/T$ , 11 of the 21 terms drop out and we are left to compute in order ten to generate the probability of bit error. If we name these ten terms (“clusters of summations”)  $\text{SUM}_1, \text{SUM}_2, \dots$ , and  $\text{SUM}_{10}$ , then for  $L_0 = 1$ ,  $L = 0$ ,  $\phi_k = 0$ , and  $\omega_k = 2\pi kI/T$

$$\int_0^T |s(t)|^2 dt = \text{SUM}_1 + \text{SUM}_2 + \cdots + \text{SUM}_{10} \quad (256)$$

and using Eq. (255)

$$X = \mathcal{R}(\text{SUM}_1 + \text{SUM}_2 + \cdots + \text{SUM}_{10}) \quad (257)$$



Now  $PT$  may be factored out of each sum. So we define

$$\text{SUM}_j = PT(\text{QUOT}_j) \quad j = 1, \dots, 10 \quad (258)$$

So

$$X = \mathcal{R} PT(\text{QUOT}_1) + \mathcal{R} PT(\text{QUOT}_2) + \dots + \mathcal{R} PT(\text{QUOT}_{10}) \quad (259)$$

and

$$X = \mathcal{R} PT[\text{QUOT}_1 + \text{QUOT}_2 + \dots + \text{QUOT}_{10}] \quad (260)$$

Also, letting  $\text{QUOT}_1 + \text{QUOT}_2 + \dots + \text{QUOT}_{10} = \text{QUOTSUM}$ , we also see

$$X = \mathcal{R} PT[\text{QUOTSUM}] \quad (261)$$

We can also say that, in general,  $\text{QUOT}_1, \text{QUOT}_2, \dots, \text{QUOT}_{10}$ , and  $\text{QUOTSUM}$  are dependent functions of  $\psi_p$  and  $b_{0,0}$ . Thus, we may write in general

$$\text{QUOT}_1(\psi_p, b_{0,0}), \text{QUOT}_2(\psi_p, b_{0,0}), \dots, \text{QUOT}_{10}(\psi_p, b_{0,0})$$

and

$$\text{QUOTSUM}(\psi_p, b_{0,0})$$

Now we can see that

$$X_1(\psi_p) = \mathcal{R} PT[\text{QUOTSUM}(\psi_p, b_{0,0} = 1)] \quad (262)$$

$$X_0(\psi_p) = \mathcal{R} PT[\text{QUOTSUM}(\psi_p, b_{0,0} = 0)] \quad (263)$$

Recalling that

$$V_T = \frac{X_0 \text{ max} + X_1 \text{ min}}{2}$$

we let

$$\begin{aligned} \psi_p \text{ max} &= \text{Value of } \psi_p \text{ which causes maximum value of} \\ &\text{of } X_0 \text{ (or maximum value of } X \text{ with } b_{0,0} = 0). \end{aligned} \quad (264)$$

and

$$\begin{aligned} \psi_{p \min} = & \text{Value of } \psi_p \text{ which causes minimum value of} \\ & \text{of } X_1 \text{ (or minimum value of } X \text{ with } b_{0,0} = 1). \end{aligned} \quad (265)$$

Then using Eqs. (249), (262)–(265)

$$V_T = \frac{\mathcal{R}PT[\text{QUOTSUM}(\psi_{p \max}, b_{0,0} = 0) + \text{QUOTSUM}(\psi_{p \min}, b_{0,0} = 1)]}{2} \quad (266)$$

Now using Eqs. (246), (262), (263), and (266)

$$P(\text{error}/\psi_p) =$$

$$\begin{aligned} & \frac{1}{2}Q \left( \left( \frac{\mathcal{R}PT}{\sqrt{N_0T}} \right) \left( [\text{QUOTSUM}(\psi_p, b_{0,0} = 1)] \right. \right. \\ & \left. \left. - \left[ \frac{\text{QUOTSUM}(\psi_{p \max}, b_{0,0} = 0) + \text{QUOTSUM}(\psi_{p \min}, b_{0,0} = 1)}{2} \right] \right) \right) \\ & + \frac{1}{2}Q \left( \left( \frac{\mathcal{R}PT}{\sqrt{N_0T}} \right) \left( \left[ \frac{\text{QUOTSUM}(\psi_{p \max}, b_{0,0} = 0) + \text{QUOTSUM}(\psi_{p \min}, b_{0,0} = 1)}{2} \right] \right. \right. \\ & \left. \left. - [\text{QUOTSUM}(\psi_p, b_{0,0} = 0)] \right) \right) \end{aligned} \quad (267)$$

Thus, we see that  $P(\text{error}/\psi_p)$  is directly related to the signal to noise ratio which we will call  $Z$ .

$$Z = \frac{\mathcal{R}PT}{\sqrt{N_0T}} = \mathcal{R}P\sqrt{\frac{T}{N_0}} \quad (268)$$

Thus, when we compute a probability of the bit error graph, we will choose a suitable range of values for  $Z$ , a value of free spectral range-bit period product  $\beta T$ , and several values of the number of adjacent channels  $M$ . For each of these values of  $M$ , we will compute a probability of bit error trace. To compute a point on a trace, choose a value of  $Z$ , compute all eight values of  $P(\text{error}/\psi_p)$ , sum all eight values of  $P(\text{error}/\psi_p)$ , and divide by 8 [see Eqs. (248), (254), and (267)]. The point is then plotted.

Recall that

$$q = \frac{T}{\frac{1}{\beta}} \quad (269a)$$

then

$$q = \beta T \quad (269b)$$

We will use this and the fact that [see Eq. (259)]

$$X = \underbrace{\mathcal{R} PT(\text{QUOT}_1)}_{\text{Term \#1}} + \underbrace{\mathcal{R} PT(\text{QUOT}_2)}_{\text{Term \#2}} + \cdots + \underbrace{\mathcal{R} PT(\text{QUOT}_{10})}_{\text{Term \#10}} \quad (270)$$

to compute the probability of bit error graphs. We also use Eqs. (256), (257), (258), (260), (261), (262), (263), (264), (265), (266), and (267) in this endeavor.

We now proceed to write out the ten terms of  $X$ , which we will use to compute the probability of bit error graphs. Before we begin we recall that  $\max(x_1, x_2)$  means choose the largest of  $x_1$  or  $x_2$ . If  $x_2 = x_1$ , then  $\max(x_1, x_2) = x_1 = x_2$ . Also recall that  $\min(x_1, x_2)$  means choose the smallest of  $x_1$  or  $x_2$ . If  $x_1 = x_2$ , then  $\min(x_1, x_2) = x_1 = x_2$ . The first term to be programmed will be Eq. (218), Appendix A. Factoring out a  $T$  we get

$$\text{SUM}_1 = PT(1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{q-1} \sum_{m=0}^{q-1} \rho^{g+m} \left( 1 - \left( \frac{\max(g, m)}{\beta T} \right) \right) \quad (271)$$

Using  $q = \beta T$  and multiplying by  $\mathcal{R}$  we get

$$\text{Term \#1} = \mathcal{R} PT(1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \left( 1 - \left( \frac{\max(g, m)}{\beta T} \right) \right) \quad (272)$$

With  $L_0 = 1$  and  $q = \beta T$ , the first term ("cluster of summations") in Eq. (219), Appendix A, becomes

$$P(1 - \rho)^2 \sum_{g=1}^{\beta T-1} \sum_{m=1}^{\beta T-1} (b_{0,-1})^2 \left( \frac{1}{\beta} \min(g, m) \right) \quad (273)$$

Factoring out a  $T$  and multiplying by  $\mathcal{R}$  we get

$$\text{Term \#2} = \mathcal{R} PT(1 - \rho)^2 (b_{0,-1})^2 \sum_{g=1}^{\beta T-1} \sum_{m=1}^{\beta T-1} \rho^{g+m} \frac{1}{\beta T} \min(g, m) \quad (274)$$

With  $L_0 = 1$  and  $q = \beta T$ , the second term (“cluster of summations”) in Eq. (219), Appendix A, becomes

$$2P(1-\rho)^2 \sum_{g=1}^{\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} (b_{0,-1})^2 G_1(g, i = -1, m, r = -1) \quad (275)$$

where

$$G_1(g, i = -1, m, r = -1) = \begin{cases} \frac{1}{\beta} [g - (m - \beta T)], & \text{for } m - \beta T < g \\ 0, & \text{otherwise} \end{cases}$$

Factoring a  $T$  and multiplying by  $\mathcal{R}$ , we arrive at the expression for Term #3

$$\text{Term \#3} = \mathcal{R} PT \times 2(1-\rho)^2 (b_{0,-1})^2 \sum_{g=1}^{\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} \left[ \frac{1}{T} \times G_1(g, i = -1, m, r = -1) \right] \quad (276)$$

With  $L_0 = 1$  and  $q = \beta T$ , the third term (“cluster of summations”) in Eq. (219), Appendix A, yields Term #4, after factoring a  $T$  and multiplying by  $\mathcal{R}$ .

$$\text{Term \#4} = \mathcal{R} PT (1-\rho)^2 (b_{0,-1})^2 \sum_{g=\beta T}^{2\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} \left[ 1 - \frac{1}{\beta T} \max(g - \beta T, m - \beta T) \right] \quad (277)$$

The next applicable term is Eq. (221) of Appendix A. Our given parameters include  $\phi_k = \phi_n = 0$ . We can then say by recalling that  $b_{k,\ell} \in \{0, e^{j\phi_k}\}$

$$b_{n,0}^* = b_{n,0} \quad \text{and} \quad b_{k,0}^* = b_{k,0} \quad (278)$$

We will also use the facts

$$\omega_k = \frac{2\pi k I}{T} \quad \text{and} \quad \omega_n = \frac{2\pi n I}{T} \quad (279)$$

where  $I$  is a positive integer called the normalized channel spacing integer. Now, by slightly reworking the  $\omega_k \neq \omega_n$  expression by substituting the results given in Eqs. (278) and (279) into the appropriate places, we get

$$\begin{aligned} \varphi_1 = & b_{k,0} b_{n,0} \frac{T}{j[2\pi I(k-n)]} \left[ e^{j[2\pi I(k-n) - (2\pi k I(g)/\beta T) + (2\pi n I(m)/\beta T)]} \right. \\ & \left. - e^{j[(2\pi I/\beta T)(k-n) \max(g,m) - (2\pi k I(g)/\beta T) + (2\pi n I(m)/\beta T)]} \right], \quad \text{for } k \neq n \quad (280) \end{aligned}$$

Note: The  $2\pi I(k - n)$  term in the first phasor of Eq. (280) contributes integer multiples of  $2\pi$  to the phasor, and so it is neglected in the next rewrite of the term. Then, by factoring the common factor  $e^{j(2\pi I/\beta T)[(nm)-(kg)]}$ , we arrive at

$$\varphi_1 = \frac{Tb_{k,0}b_{n,0}e^{j(2\pi I/\beta T)[(nm)-(kg)]}}{j[2\pi I(k - n)]} \left[ 1 - e^{j(2\pi I/\beta T)(k-n)\max(g,m)} \right], \quad \text{for } k \neq n \quad (281)$$

Finally, after we factor out a  $T$  from both the  $\omega_k \neq \omega_n$  and  $\omega_k = \omega_n$  arguments to the summations, rearrange the summations for more logical programming, and use the fact that  $q = \beta T$ , we arrive at the final form for Term #5.

$$\text{Term \#5} = \mathcal{R} PT(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{\substack{n=-M/2 \\ n \neq 0}}^{+M/2} (b_{k,0})(b_{n,0}) \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \times \varphi_1 \quad (282)$$

where

$$\varphi_1 = \begin{cases} \frac{e^{j(2\pi I/\beta T)[(nm)-(kg)]}}{j[2\pi I(k - n)]} \left[ 1 - e^{j(2\pi I/\beta T)(k-n)\max(g,m)} \right] & \text{for } k \neq n \\ e^{j[(2\pi k I/\beta T)(m-g)]} \left[ 1 - \frac{\max(g,m)}{\beta T} \right], & \text{for } k = n \end{cases}$$

The next applicable term is the first term ("cluster of summations") in Eq. (227), Appendix A. Utilizing  $L_0 = 1$ , factoring a  $T$ , multiplying by  $\mathcal{R}$ , and noting that  $q = \beta T$  yields

$$\text{Term \#6} = \mathcal{R} PT \times 2(1 - \rho)^2 (b_{0,0})(b_{0,-1}) \sum_{g=0}^{\beta T-1} \sum_{m=1}^{\beta T-1} \rho^{g+m} \frac{G_4(g, i = -1, m)}{T} \quad (283)$$

where

$$\frac{G_4(g, i = -1, m)}{T} = \begin{cases} \frac{1}{\beta T} [m - g], & \text{for } g < m \\ 0, & \text{otherwise} \end{cases}$$

Looking at the second term ("cluster of summations") in Eq. (227), Appendix A, and after utilizing  $L_0 = 1$ , factoring a  $T$ , multiplying by  $\mathcal{R}$ , and substituting

$q = \beta T$ , we have

$$\text{Term \#7} = \mathcal{R} PT \times 2(1 - \rho)^2 (b_{0,0})(b_{0,-1}) \sum_{g=0}^{\beta T-1} \sum_{m=\beta T}^{2\beta T-1} \rho^{g+m} \left[ 1 - \frac{1}{\beta T} \max(g, m - \beta T) \right] \quad (284)$$

The next applicable term is the first term (“cluster of summations”) in Eq. (228), Appendix A. Since  $\phi_k = 0$

$$b_{k,0}^* = b_{k,0} \quad (285)$$

We can then work with the term inside of the summations

$$\begin{aligned} \varphi_7 = & \rho^{g+m} b_{0,0} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k(T-(m/\beta))} - e^{j\omega_k((\max(g,m)-m)/\beta)} \right] \\ & + \rho^{g+m} b_{0,0} b_{k,0}^* \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k(T-(m/\beta))} - e^{-j\omega_k((\max(g,m)-m)/\beta)} \right] \end{aligned} \quad (286)$$

Then, since  $b_{k,0}^* = b_{k,0}$  and after some rearranging, we have

$$\begin{aligned} \varphi_7 = & \frac{\rho^{g+m} b_{0,0} b_{k,0}}{\omega_k} \times \dots \\ & \left[ \frac{e^{j\omega_k(T-(m/\beta))} - e^{-j\omega_k(T-(m/\beta))}}{j} - \underbrace{\frac{e^{j\omega_k((\max(g,m)-m)/\beta)} + e^{-j\omega_k((\max(g,m)-m)/\beta)}}{j}}_{\Lambda} \right] \end{aligned} \quad (287)$$

Then factoring a  $-1$  from  $\Lambda$  yields

$$\Lambda = - \left( \frac{e^{j\omega_k((\max(g,m)-m)/\beta)} - e^{-j\omega_k((\max(g,m)-m)/\beta)}}{j} \right) \quad (288)$$

Euler’s Relationship defines

$$\sin \theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad (289)$$

Applying this relationship yields

$$\varphi_7 = \frac{2\rho^{g+m} b_{0,0} b_{k,0}}{\omega_k} \left[ \sin \left( \omega_k \left( T - \frac{m}{\beta} \right) \right) - \sin \left( \omega_k \left( \frac{\max(g, m) - m}{\beta} \right) \right) \right] \quad (290)$$

and since  $\omega_k = 2\pi kI/T$

$$\varphi_7 = 2 \frac{\rho^{g+m} b_{0,0} b_{k,0}}{\left(\frac{2\pi kI}{T}\right)} \left[ \sin \left[ 2\pi kI \left( 1 - \frac{m}{\beta T} \right) \right] - \sin \left[ \frac{2\pi kI}{\beta T} (\max(g, m) - m) \right] \right] \quad (291)$$

After factoring a  $T$ , multiplying by  $\mathcal{R}$ , utilizing  $q = \beta T$ , and rearranging the summations for more logical programming, we arrive at the expression for Term #8.

$$\text{Term \#8} = \mathcal{R} PT(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \varphi_7$$

where

$$\varphi_7 = \frac{\rho^{g+m} b_{0,0} b_{k,0}}{\pi kI} \left[ \sin \left[ 2\pi kI \left( 1 - \frac{m}{\beta T} \right) \right] - \sin \left[ \frac{2\pi kI}{\beta T} (\max(g, m) - m) \right] \right] \quad (292)$$

The next applicable term ("cluster of summations") is Eq. (229) of Appendix A. Since  $\phi_k = 0$ , we know  $b_{k,0} = b_{k,0}^*$ . Using  $L_0 = 1$ , we can rework the argument of the summations. Making the appropriate parameter substitutions yields

$$\begin{aligned} \varphi_{10} = & \rho^{g+m} b_{0,-1} b_{k,0} \frac{1}{j\omega_k} \left[ e^{j\omega_k[(g-m)/\beta]} - 1 \right] G_6(g, i = -1, m) \\ & + \rho^{g+m} b_{0,-1} b_{k,0} \frac{1}{[-j\omega_k]} \left[ e^{-j\omega_k[(g-m)/\beta]} - 1 \right] G_6(g, i = -1, m) \end{aligned} \quad (293)$$

Further simplifying yields

$$\varphi_{10} = \frac{\rho^{g+m} (b_{0,-1})(b_{k,0})}{\omega_k} G_6(g, i = -1, m) \left[ \underbrace{\frac{e^{j\omega_k[(g-m)/\beta]} - e^{-j\omega_k[(g-m)/\beta]}}{j}}_{2 \sin[\omega_k[(g-m)/\beta]]} \right] \quad (294)$$

Using  $\omega_k = 2\pi kI/T$  yields

$$\varphi_{10} = \frac{T \rho^{g+m} (b_{0,-1})(b_{k,0})}{\pi kI} G_6(g, i = -1, m) \sin \left[ \frac{2\pi kI}{\beta T} (g - m) \right] \quad (295)$$

Substituting  $\varphi_{10}$  inside of the summations, multiplying by  $\mathcal{R}$ , factoring  $T$ , using  $q = \beta T$ , and rearranging the summations for logical programming yields the following

for Term #9.

$$\text{Term \#9} = \mathcal{R} PT(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=1}^{\beta T-1} \sum_{m=0}^{\beta T-1} \varphi_{10} \quad (296)$$

where

$$\varphi_{10} = \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi k I} G_6(g, i = -1, m) \sin \left[ \frac{2\pi k I}{\beta T} (g - m) \right]$$

and

$$G_6(g, i = -1, m) = \begin{cases} 1, & \text{for } m < g \\ 0, & \text{otherwise} \end{cases}$$

Look at the last applicable term (“cluster of summations”), Eq. (232), Appendix A. Again we take advantage of the fact that  $\phi_k = 0$ , which means  $b_{k,0} = b_{k,0}^*$ . Making the appropriate parameter substitutions yields

$$\begin{aligned} \varphi_{13} = & \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\omega_k} \frac{1}{j} \left[ e^{j\omega_k[T-(m/\beta)]} - e^{-j\omega_k[T-(m/\beta)]} \right. \\ & \left. - \left[ e^{j\omega_k[(1/\beta)\max(g-\beta T, m)-m/\beta]} - e^{-j\omega_k[(1/\beta)\max(g-\beta T, m)-m/\beta]} \right] \right] \end{aligned} \quad (297)$$

Using Euler’s Relationship we get

$$\varphi_{13} = \frac{T\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi k I} \left[ \sin \left[ 2\pi k I \left( 1 - \frac{m}{\beta T} \right) \right] - \sin \left[ \frac{2\pi k I}{\beta T} (\max(g - \beta T, m) - m) \right] \right] \quad (298)$$

Substituting  $\varphi_{13}$  back inside of the summations, factoring a  $T$ , multiplying by  $\mathcal{R}$ , utilizing  $q = \beta T$ , and rearranging the summations for logical programming yields

$$\text{Term \#10} = \mathcal{R} PT(1 - \rho)^2 \sum_{\substack{k=-M/2 \\ k \neq 0}}^{+M/2} \sum_{g=\beta T}^{2\beta T-1} \sum_{m=0}^{\beta T-1} \varphi_{13} \quad (299)$$

where

$$\varphi_{13} = \frac{\rho^{g+m}(b_{0,-1})(b_{k,0})}{\pi k I} \left[ \sin \left[ 2\pi k I \left( 1 - \frac{m}{\beta T} \right) \right] - \sin \left[ \frac{2\pi k I}{\beta T} (\max(g - \beta T, m) - m) \right] \right]$$



For the given parameters  $L_0 = 1$ ,  $L = 0$ ,  $\phi_k = 0$ , and  $\omega_k = 2\pi kI/T$ , we have now derived all ten programmable terms of  $X$  where

$$X = \underbrace{\mathcal{R} PT(\text{QUOT}_1)}_{\text{Term \#1}} + \underbrace{\mathcal{R} PT(\text{QUOT}_2)}_{\text{Term \#2}} + \cdots + \underbrace{\mathcal{R} PT(\text{QUOT}_{10})}_{\text{Term \#10}} \quad (300)$$

We will use these ten programmable terms to compute probability of bit error for the dense WDM system.



## APPENDIX C

### PROGRAMMING STRATEGY AND COMPUTER PROGRAMS FOR GENERATION OF PROBABILITY OF BIT ERROR GRAPHS

In this appendix, we present the computer programs written to generate the probability of bit error graphs for the system under consideration. We attempt to generate four graphs, one for each value of  $\beta T$  considered. We reiterate that  $\beta$  is the free spectral range of the Fabry-Perot filter (Hz) and that  $T$  is the data bit period (s).  $\beta T$  is called the free spectral range-bit period product. The four values of  $\beta T$  used are

$$\beta T = [500 \quad 1000 \quad 1500 \quad 2000]$$

In each of the four graphs there will be five traces as we will present probability bit error for four values of the normalized channel spacing integer  $I$ , or equivalently the number of adjacent channels  $M$ , along with a probability of bit error trace for single channel (SC) operation without Fabry-Perot (FP) filtering or SC operation with FP filtering and without ISI or ACI. We will show the relationship between  $I$  and the number of adjacent channels  $M$  later. Now, however, we present the four values of  $I$  corresponding to each value of  $\beta T$ :

For  $\beta T = 500$

$$I = [4 \quad 5 \quad 8 \quad 20]$$

For  $\beta T = 1000$

$$I = [5 \quad 6 \quad 9 \quad 20]$$

For  $\beta T = 1500$

$$I = [7 \quad 9 \quad 12 \quad 20]$$

For  $\beta T = 2000$

$$I = [8 \quad 9 \quad 12 \quad 20]$$

Without a lengthy discourse, we present the mathematical relationship between the normalized channel spacing integer  $I$  and the number of adjacent channels  $M$

$$M = \frac{\beta}{\Delta f} - 1 \quad (301)$$

For the special case  $f_k = k\Delta f = kI/T$  we can easily see that  $\Delta f = I/T$ . Then

$$M = \frac{\beta T}{I} - 1 \quad (302)$$

For the values of  $I$  above  $M$  will not always be an even integer. Thus, an algorithm to consistently arrive at an even integer value of  $M$  which is less than or equal to the true mathematical value presented in Eq. (302) was devised. To get the number of adjacent channels, we perform the following operation

$$\frac{\beta T}{I} = Q + R \quad (303)$$

where  $Q$  is the integer quotient of the division operation and  $R$  is the remainder. To arrive at  $M$  we

1. Subtract 1 from  $Q$  if  $Q$  is an odd integer and  $R = 0$ .
2. Subtract 2 from  $Q$  if  $Q$  is an even integer and  $R = 0$ .
3. Subtract 1 from  $Q$  if  $Q$  is an odd integer and  $R \neq 0$ .
4. Subtract 2 from  $Q$  if  $Q$  is an even integer and  $R \neq 0$ .

Using these rules, we obtain at the four values of  $M$  for each value of  $\beta T$ .

For  $\beta T = 500$

$$M = [\underbrace{124}_{I=4} \quad \underbrace{98}_{I=5} \quad \underbrace{60}_{I=8} \quad \underbrace{24}_{I=20}]$$

For  $\beta T = 1000$

$$M = \left[ \underbrace{198}_{I=5} \quad \underbrace{164}_{I=6} \quad \underbrace{110}_{I=9} \quad \underbrace{48}_{I=20} \right]$$

For  $\beta T = 1500$

$$M = \left[ \underbrace{212}_{I=7} \quad \underbrace{164}_{I=9} \quad \underbrace{124}_{I=12} \quad \underbrace{74}_{I=20} \right]$$

For  $\beta T = 2000$

$$M = \left[ \underbrace{248}_{I=8} \quad \underbrace{220}_{I=9} \quad \underbrace{164}_{I=12} \quad \underbrace{98}_{I=20} \right]$$

Now we use these four values of  $I$  and four values of  $M$  for each value of  $\beta T$  to compute the four graphs of probability of bit error with five traces each. We will use the equations and methods developed in Appendix B to compute these graphs. Recall that

$$X = \underbrace{\mathcal{R} PT(\text{QUOT}_1)}_{\text{Term \#1}} + \underbrace{\mathcal{R} PT(\text{QUOT}_2)}_{\text{Term \#2}} + \cdots + \underbrace{\mathcal{R} PT(\text{QUOT}_{10})}_{\text{Term \#10}} \quad (304)$$

Realizing this, we program the ten terms given in Appendix B in the following way:

TERM #1 [Eq. (272)]: We will compute the QUOT<sub>1</sub> portion of this term four times, once for each value of  $\beta T = 500, 1000, 1500$ , and  $2000$ . We will use each of these four values in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #2 [Eq. (274)]: We will compute the QUOT<sub>2</sub> portion of TERM #2 four times, once for each value of  $\beta T = 500, 1000, 1500$ , and  $2000$ . Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #3 [Eq. (276)]: We will compute the QUOT<sub>3</sub> portion of TERM #3 four times, once each value of  $\beta T = 500, 1000, 1500$ , and  $2000$ . Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #4 [Eq. (277)]: We will compute the  $QUOT_4$  portion of TERM #4 four times, once for each value of  $\beta T = 500, 1000, 1500,$  and  $2000$ . Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #5 [Eq. (282)]: We will compute the value of  $QUOT_5$  a total of 48 times, 12 times for each value of  $\beta T = 500, 1000, 1500,$  and  $2000$ . For each value of  $\beta T$ , we have the four corresponding values of the normalized channel spacing integer  $I$ , or equivalently, the number of adjacent channels  $M$ . We will compute the value of  $QUOT_5$  three times for each value of  $I$ .  $QUOT_5$  will be computed once for all  $M/2$  of the lower adjacent channels being packed with 1s and the upper adjacent channels being packed with 0s. This is the case:  $(b^- = 1, b^+ = 0)$ .  $QUOT_5$  will be computed once for the lower adjacent channels being packed with 0s and the upper adjacent channels being packed with 1s. This is the case:  $(b^- = 0, b^+ = 1)$ . Finally, we will compute  $QUOT_5$  once more for both the lower and upper adjacent channels being packed with 1's. This is the case:  $(b^- = 1, b^+ = 1)$ . These twelve values of  $QUOT_5$  for each value of  $\beta T$  will be used in each of the four separate bit error programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

TERM #6 [Eq. (283)]: We will compute the  $QUOT_6$  portion of TERM #6 four times, once each for  $\beta T = 500, 1000, 1500,$  and  $2000$ . Each of the values will be used in separate to compute program probability of bit error according to the equations developed in Appendix B.

TERM #7 [Eq. (284)]: We will compute the  $QUOT_7$  portion of TERM #7 four times, once for each value of  $\beta T = 500, 1000, 1500,$  and  $2000$ . Each of these values will be used in a separate program to compute the probability of bit error according to the equations developed in Appendix B.

TERM #8 [Eq. (292)]: We compute the value of  $QUOT_8$  twelve times for each value of  $\beta T$ . These twelve values of  $QUOT_8$  for each value of  $\beta T$  will be used in each of the four separate bit error graphing programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

TERM #9 [Eq. (296)]: We compute the value of  $QUOT_9$  twelve times for each value of  $\beta T$ . These twelve values of  $QUOT_9$  for each value of  $\beta T$  will be used in each of the four separate bit error graphing programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

TERM #10 [Eq. (299)]: We compute the value of  $QUOT_{10}$  twelve times for each value of  $\beta T$ . These twelve values of  $QUOT_{10}$  for each value of  $\beta T$  will be used in each of the four separate bit error graphing programs to compute each of the four multiple channel probability of bit error traces using the equations and methods developed in Appendix B.

*Note:* When we say we are computing  $QUOT_j$ , we are not being exactly mathematically correct, as we compute each of these terms without the bit values  $b_{0,0}$  and/or  $b_{0,-1}$  the factor  $(1 - \rho)^2$ . These are accounted for in the final programs. For example

$$\text{Term \#1} = \underbrace{\mathcal{R}PT (1 - \rho)^2 b_{0,0}^2 \sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \left( 1 - \left( \frac{\max(g, m)}{\beta T} \right) \right)}_{QUOT_1} \quad (305)$$

However, the computer program for Term #1 (QUOT<sub>1</sub>) only computes

$$\sum_{g=0}^{\beta T-1} \sum_{m=0}^{\beta T-1} \rho^{g+m} \left( 1 - \left( \frac{\max(g, m)}{\beta T} \right) \right) \quad (306)$$

four times for  $\beta T = 500, 1000, 1500, 2000$ .  $(1 - \rho)^2$  and the value of  $(b_{0,0})^2$  are accounted for in the final programs which utilize the equations and methods developed in Appendix B. The author apologizes for the slight stretch of the truth, but it seemed necessary to succinctly explain the general method of computing each of the ten terms, and the final graphs.

For completeness we also present the probability of bit error equation for a single channel operation

$$P_b = Q\left(\frac{1}{2}Z\right) \quad (307)$$

where we recall [Eq. (271), Appendix B] that

$$Z = \frac{\mathcal{R} PT}{\sqrt{N_0 T}} = \mathcal{R} P \sqrt{\frac{T}{N_0}} \quad (308)$$

and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy \quad (309)$$

The computer programs for each of the ten terms, the numerical results, and the final graph are now presented.



```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%                               THESIS COMPUTER WORK
%
%                               |-----|
%                               | TERM #1 |
%                               |-----|
%
%  COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES OF
%             TERM #1 AND TO WRITE THESE VALUES TO DIARY FILE FOR LATER
%             USE/MANIPULATION IN CALCULATING THE DETECTION STATISTIC
%             AND THE PROBABILITIES OF BIT ERROR
%
%  JOHN A. STUDER                               DATE LAST MODIFIED: 11 SEP 94
%  CPT, U.S ARMY
%  550-53-7181
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

```

```

rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
    term1sum(i)= 0;
    for g = 1:betatau(i)
        for m = 1:betatau(i)
            term1sum(i) = term1sum(i)+(rho.^((g-1)+(m-1))*...
                (1-(max([g-1 m-1]'))/betatau(i))));
        end
    end
end
diary johnman1.txt
diary on
term1sum
diary off
end

```

```

term1sum =
    1.0e+03 *
    7.0510    8.5126    9.0083    9.2563    % numbers/arrows/words
                                                % below added after matlab
                                                % dumped answers to file using
                                                % the diary command

    ^         ^         ^         ^
    500       1000      1500      2000 ---- betatau values

% The numbers displayed above are the computed values of
% Term #1 for the four values of betatau given above.
% These values will be used later in other programs to compute detection
% statistics and the probabilities of bit error for various signal to noise
% ratios.

```

TERM #1
---------

%%%

THESIS COMPUTER WORK

TERM #2

COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE THE FOUR VALUES  
OF TERM #2 CORRESPONDING TO THE FOUR VALUES OF  
betatau: 500,1000,1500,2000. WE WILL USE BACKGROUND  
PROCESSING TO WRITE THESE FOUR VALUES TO AN OUTPUT  
FILE CALLED term2.out. WE WILL USE THESE FOUR OUTPUT  
VALUES FOR TERM #2 TO LATER, IN ANOTHER PROGRAM  
COMPUTE THE DETECTION STATISTICS AND PROBABILITIES  
OF BIT ERROR FOR THE VARIOUS SIGNAL TO NOISE RATIOS.

JOHN A. STUDER DATE LAST MODIFIED: 11 SEP 94  
CPT, U.S. ARMY  
550-53-7181

%%%

```
rho = 0.99;
betatau = [500 1000 1500 2000];
for i = 1:4
    term2sum(i) = 0;
    for g = 1:betatau(i)-1
        for m = 1: betatau(i)-1
            term2sum(i) = term2sum(i) + ((rho^(g+m))*min(g,m));
        end
    end
    term2sumfinal(i) = term2sum(i)/betatau(i);
end
term2sumfinal
exit
```

< M A T L A B (R) >  
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 All Rights Reserved  
 Version 4.2  
 Mar 29 1994

Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

term2sumfinal =		% The numbers/arrows/words
		% below were added after
959.5662	492.4271	% the matlab background job
		% dumped the values to the
		% file term2.out
%     ^	%     ^	% All values/text not done by
%     500	%     1000	% Sun Stn is preceded by a "%"
%     ^	%     ^	
%     1500	%     2000	
%     ^		
%     ----- betatau values-----		

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                               %
%   TERM #2                     %
%                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  
```

29965020 flops.



```
Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe
```

```
term3sumfinal = % Numbers/arrows/words
                % below were added after
    1.2211e+01    4.2917e-02    1.8815e-04    9.2720e-07 % the matlab background
                % job dumped the values
    %      ^           ^           ^           ^         % to the file term3.out
    %      500        1000        1500        2000      % All values/text not
    %                                                    % done by Sun Stn. is
    %                                                    % preceded by a "%".
    % ----- betatau values -----
```

```

%%%%%%%%%%%%%%
%                                     %
%      TERM #3                      %
%                                     %
%%%%%%%%%%%%%%

```

132



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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

term4sumfinal =					% Numbers/arrows/words
					% below were added after
3.0440e-01	1.5865e-05	7.2482e-10	3.2152e-14		% the matlab bkgd job
					% dumped the values to
% ^	% ^	% ^	% ^		% the file term4.out
% 500	% 1000	% 1500	% 2000		% All values/text not
%					% done by Sun Stn is
%					% preceded by a "%".
%	----- betatau values -----				

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%
%   TERM #4
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  
```

60010008 flops.





```

A = colprime-rowprime;    % m-g for later complex exponential
for i = 1:4
    bitmatrix = [-M(i)/2:-1 zeros(1,M(i)/2);zeros(1,M(i)/2) 1:M(i)/2;...
                -M(i)/2:-1 1:M(i)/2];
    const = ((2*pi*I(i))/betatau);
    for bitpat = 1:3
        term5sum(i,bitpat) = 0;
        work = bitmatrix(bitpat,:);
        m = work==0;
        work(m)=[];
        k = work;
        n = k;
        for kct = 1:length(k)
            for nct = 1:length(n)
                if kct == nct
                    B = j*((2*pi*k(kct)*I(i)*A)/betatau);
                    D = exp(B);
                    term5sum(i,bitpat) = term5sum(i,bitpat)+...
                        sum(sum(rhomatrix.*D.*ONEMINUSMAX));
                else
                    E1 = exp(j*const*((n(nct)*colprime)-(k(kct)*rowprime)));%%%
                    E2 = 1-exp(j*const*(k(kct)-n(nct))*W0);
                    term5sum(i,bitpat) = term5sum(i,bitpat) +...
                        sum(sum((rhomatrix.*E1.*E2)/(j*2*pi*I(i)*(k(kct)-n(nct)))));
                end
            end
        end
    end
end
end
save term5500 term5sum
exit

```

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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

```
>> load term5500.mat
>> who
```

Your variables are:

term5sum

```
>> term5sum
```

term5sum =

1.0e+03 \*

1.5423 - 0.0000i	1.5423 + 0.0000i	1.3825 + 0.0000i	% I = 4
0.9419 - 0.0000i	0.9419 + 0.0000i	0.9001 - 0.0000i	% I = 5
0.3307 - 0.0000i	0.3307 + 0.0000i	0.3576 + 0.0000i	% I = 8
0.0431 - 0.0000i	0.0431 + 0.0000i	0.0580 - 0.0000i	% I = 20

```
>>
```

```
%
```

```
% CASE: (b-=1,b+=0) CASE: (b-=0,b+=1) CASE: (b-=1,b+=1)
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%          TERM #5          %
%  betatau = 500  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term5500.out CREATED FROM THE  
 BINARY FILE term5500.mat USING MATLAB INTERACTIVE COMMANDS.  
 ALL TEXT PRECEDED BY A % WAS ADDED LATER USING A  
 TEXT EDITOR.



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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

term6sumfinal =					% Numbers/arrows/words
					% below were added after
1.8585e+03	9.9411e+02	6.6331e+02	4.9749e+02		% the matlab bkgd job
					% dumped the values to
% ^	% ^	% ^	% ^		% the file term6.out
% 500	% 1000	% 1500	% 2000		% All values/text not
%					% done by Sun Stn is
%	-----	betatau values	-----		% preceded by a "%".

```

%*****%
%      %
%  TERM #6  %
%      %
%*****%
  
```

33732508 flops.



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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

term7sumfinal =					% Numbers/arrows/words
					% below were added after
9.2657e+01	7.3500e-01	5.1105e-03	3.4503e-05		% the matlab bkgd job
					% dumped the values to
% ^	% ^	% ^	% ^		% the file term7.out
% 500	% 1000	% 1500	% 2000		% All values/text not
%					% done by Sun Stn is
%					% preceded by a "%".
%					
%	----- betatau values -----				

```

XXXXXXXXXXXXXXXXX
%                %
%   TERM #7     %
%                %
XXXXXXXXXXXXXXXXX
  
```

67510004 flops.





```
        term8500left(i) = term8sum500(i);
    end
end
term8500lr(i) = term8sum500(i);
term8500right(i) = term8500lr(i)-term8500left(i);
end
term8500left
term8500right
term8500lr
exit
```

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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

term8500left =

-3.7098e+02 -2.3852e+02 -9.2820e+01 -1.4109e+01 % CASE: (b-=1,b+=0)

term8500right =

-3.7098e+02 -2.3852e+02 -9.2820e+01 -1.4109e+01 % CASE: (b-=0,b+=1)

term8500lr =

-7.4196e+02 -4.7703e+02 -1.8564e+02 -2.8217e+01 % CASE: (b-=1,b+=1)

```
%      ^           ^           ^           ^
%      I = 4       I = 5       I = 8       I = 20
%      -----
%      ^
%      VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #8      %
%      betatau = 500 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: ALL TEXT PRECEDED BY A % WAS ADDED  
 BY A TEXT EDITOR AFTER THE BACKGROUND  
 JOB CREATED THE FILE term8500.out.

2142000318 flops.



```

        end
    end
    if ct == M(i)/2
        term81kleft(i) = term8sum(i);
    end
end
term81klr(i) = term8sum(i);
term81kright(i) = term81klr(i)-term81kleft(i);
end
save term81k term81kleft term81kright term81klr
exit

```

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Commands to get started: intro, demo, help help  
Commands for more information: help, whatsnew, info, subscribe

```
>> load term81k
>> who
```

Your variables are:

```
term81kleft    term81klr    term81kright
```

```
>> term81kleft
```

```
term81kleft =
```

```
-464.3344 -327.7899 -148.2912 -29.7545    % CASE: (b-=1,b+=0)
```

```
>> term81kright
```

```
term81kright =
```

```
-464.3344 -327.7899 -148.2912 -29.7545    % CASE: (b-=0,b+=1)
```

```
>> term81klr
```

```
term81klr =
```

```
-928.6688 -655.5798 -296.5824 -59.5090    % CASE: (b-=1,b+=1)
```

```
>>
```

```
%      ^      ^      ^      ^
%      I = 5      I = 6      I = 9      I = 20
%      -----
%      ^
```

```
% VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #8      %
% betatau = 1000 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term81k.out CREATED FROM THE  
BINARY FILE term81k.mat USING MATLAB INTERACTIVE  
COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
USING A TEXT EDITOR.



```
        term815kleft(i) = term8sum(i);
    end
end
term815klr(i) = term8sum(i);
term815kright(i) = term815klr(i)-term815kleft(i);
end
term815kleft
term815kright
term815klr
exit
```

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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

```

malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
malloc count matches malloc_debug_count
  
```

term815kleft =

```

-3.5253e+02 -2.1854e+02 -1.2475e+02 -4.5096e+01 % CASE: (b-=1,b+=0)
  
```

term815kright =

```

-3.5253e+02 -2.1854e+02 -1.2475e+02 -4.5096e+01 % CASE: (b-=0,b+=1)
  
```

term815klr =

```

-7.0505e+02 -4.3708e+02 -2.4950e+02 -9.0192e+01 % CASE: (b-=1,b+=1)
  
```

```

%      ^           ^           ^           ^
%      I = 7       I = 9       I = 12      I = 20
%      -----
%      ^
%      VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
  
```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #8      %
%      betatau = 1500 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
  
```

NOTE: THIS IS THE FILE term815k.out. ALL TEXT  
 PRECEDED BY A % WAS ADDED LATER USING A  
 TEXT EDITOR.

36162000586 flops.





```

        end
        if ct == M(i)/2
            term82kleft(i) = term8sum(i);
        end
    end
    term82klr(i) = term8sum(i);
    term82kright(i) = term82klr(i)-term82kleft(i);
end
save term82k term82kleft term82kright term82klr
exit

```

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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

```
>> load term82k.mat
>> who
```

Your variables are:

```
term82kleft    term82klr    term82kright
```

```
>> term82kleft
```

```
term82kleft =
```

```
-351.8931 -283.0055 -163.9044 -60.1542    % CASE: (b-=1,b+=0)
```

```
>> term82kright
```

```
term82kright =
```

```
-351.8931 -283.0055 -163.9044 -60.1542    % CASE: (b-=0,b+=1)
```

```
>> term82klr
```

```
term82klr =
```

```
-703.7863 -566.0110 -327.8088 -120.3084    % CASE: (b-=1,b+=1)
```

```
%      ^           ^           ^           ^
%  I = 8       I = 9       I = 12       I = 20
%  -----
%      ^
%  VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #8      %
%  betatau = 2000 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term82k.out CREATED FROM THE  
 BINARY FILE term82k.mat USING MATLAB INTERACTIVE  
 COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
 USING A TEXT EDITOR.



```

        end
    end
end
if ct == M(i)/2
    term9500left(i) = term9sum(i);
end
end
term9500lr(i) = term9sum(i);
term9500right(i) = term9500lr(i)-term9500left(i);
end
save term9500 term9500left term9500right term9500lr
exit

```

```
<108 sp254201(SunOS) /kepler_u2/studer> matlab
```

```
      < M A T L A B (R) >
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    Jun 15 1993
```

```
Commands to get started: intro, demo, help help
Commands for more information: help, whatsnew, info, subscribe
```

```
>> load term9500.mat
>> who
```

```
Your variables are:
```

```
term9500left      term9500right
term9500lr
```

```
>> term9500left
```

```
term9500left =
```

```
    125.1608    80.4704    31.3156    4.7599    % CASE: (b-=1,b+=0)
```

```
>> term9500right
```

```
term9500right =
```

```
    125.1608    80.4704    31.3156    4.7599    % CASE: (b-=0,b+=1)
```

```
>> term9500lr
```

```
term9500lr =
```

```
    250.3216   160.9408    62.6313    9.5199    % CASE: (b-=1,b+=1)
```

```
>>
```

```
%      ^      ^      ^      ^
%      I = 4      I = 5      I = 8      I = 20
```

```
%      -----
%      ^
```

```
% VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #9      %
%      betatau = 500      %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term9500.out CREATED FROM THE BINARY  
FILE term9500.mat USING MATLAB INTERACTIVE COMMANDS. ALL  
TEXT PRECEDED BY A % WAS ADDED LATER USING A TEXT  
EDITOR.





```

        end
    end
end
if ct == M(i)/2
    term91kleft(i) = term9sum(i);
end
end
term91klr(i) = term9sum(i);
term91kright(i) = term91klr(i)-term91kleft(i);
end
save term91k term91kleft term91kright term91klr
exit

```

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Commands for more information: help, whatsnew, info, subscribe

```
>> load term91k.mat
>> who
```

Your variables are:

```
term91kleft    term91klr    term91kright
```

```
>> term91kleft
```

```
term91kleft =
```

```
    155.3047    109.6350    49.5986     9.9519    % CASE: (b-=1,b+=0)
```

```
>> term91kright
```

```
term91kright =
```

```
    155.3047    109.6350    49.5986     9.9519    % CASE: (b-=0,b+=1)
```

```
>> term91klr
```

```
term91klr =
```

```
    310.6094    219.2701    99.1971    19.9038    % CASE: (b-=1,b+=1)
```

```
>>
```

```
%      ^      ^      ^      ^
%      I = 5    I = 6    I = 9    I = 20
%      -----
%      ^
```

```
% VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #9      %
% betatau = 1000 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term91k.out CREATED FROM THE  
BINARY FILE term91k.mat USING MATLAB INTERACTIVE  
COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
USING A TEXT EDITOR.



```

        end
    end
end
if ct == M(i)/2
    term915kleft(i) = term9sum(i);
end
end
term915klr(i) = term9sum(i);
term915kright(i) = term915klr(i)-term915kleft(i);
end
save term915k term915kleft term915kright term915klr
exit

```

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```
>> load term915k
>> who
```

Your variables are:

```
term915kleft      term915kright
term915klr
```

```
>> term915kleft
```

```
term915kleft =
```

```
117.9019  73.0900  41.7221  15.0822  % CASE: (b-=1,b+=0)
```

```
>> term915kright
```

```
term915kright =
```

```
117.9019  73.0900  41.7221  15.0822  % CASE: (b-=0,b+=1)
```

```
>> term915klr
```

```
term915klr =
```

```
235.8037  146.1800  83.4441  30.1644  % CASE: (b-=1,b+=1)
```

```
%      ^      ^      ^      ^
%  I = 7      I = 9      I = 12      I = 20
%  -----
%
%  VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #9      %
%  betatau = 1500 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term915k.out CREATED FROM THE  
 BINARY FILE term915k.mat USING MATLAB INTERACTIVE  
 COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
 USING A TEXT EDITOR.

%%%

# THESIS COMPUTER WORK

```

      TERM #9
      betatau = 2000
  
```

COMMENTS: THE IDEA OF THIS PROGRAM IS TO COMPUTE TERM #9 FOR THE VALUE OF betatau = 2000. WE WILL COMPUTE FOR THE FOUR VALUES OF I (AN INTEGER). THE VALUES OF I FOR betatau = 2000 are I = [8 9 12 20]. WE WILL COMPUTE THE VALUE OF TERM #9 THREE TIMES FOR EACH CORRESPONDING VALUE OF I. ONCE FOR ALL M/2 OF THE LOWER ACI CHANNELS (have negative summation index) BEING PACKED WITH 1'S AND THE UPPER ACI CHANNELS ALL BEING PACKED WITH 0'S. THIS IS THE CASE: (b-=1,b+=0). ONCE FOR ALL THE LOWER ACI CHANNELS BEING PACKED WITH ZERO'S AND THE UPPER ACI CHANNELS (have a positive summation index) BEING PACKED WITH 1'S. THIS IS THE CASE: (b-=0,b+=1). FINALLY ONCE FOR BOTH THE UPPER AND LOWER ACI CHANNELS BEING PACKED WITH 1'S. THIS IS THE CASE: (b-=1,b+=1). WE WILL USE BACKGROUND PROCESSING WHICH WILL DUMP ALL TWELVE VALUES TO A FILE CALLED term92k.mat. SINCE THE FILE IS IN A BINARY FORMAT WE WILL USE MATLAB INTERACTIVE COMMANDS AND THE TEXT EDITOR TO PRODUCE READABLE HARDCOPY RESULTS. AFTER THIS IS ALL DONE WE WILL USE THESE VALUES LATER, IN ANOTHER PROGRAM TO COMPUTE THE DETECTION STATISTICS AND PROBABILITIES OF BIT ERROR FOR THE VARIOUS SIGNAL TO NOISE RATIOS.

JOHN A. STUDER  
CPT, U.S. ARMY  
550-53-7181

DATE LAST MODIFIED: 28 SEP 94

%%%

```

format short e
rho = 0.99;
betatau = 1500;
I = [8 9 12 20];
M = [248 220 164 98];
for i = 1:4
    k = [-M(i)/2:-1 1:M(i)/2];
    term9sum(i) = 0;
    for ct = 1:M(i)
        for g = 1:betatau-1
            for m = 1:betatau
                if m-1 < g
                    term9sum(i) = term9sum(i) + ((rho^(g+(m-1)))/(pi*k(ct)...
                        *I(i)))*sin(2*pi*k(ct)*I(i)*((g-(m-1))...
                        /betatau));
                end
            end
        end
    end
end
  
```

```

        end
    end
end
if ct == M(i)/2
    term92kleft(i) = term9sum(i);
end
end
term92klr(i) = term9sum(i);
term92kright(i) = term92klr(i)-term92kleft(i);
end
save term92k term92kleft term92kright term92klr
exit

```

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Commands to get started: intro, demo, help help  
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```
>> load term92k.mat
>> who
```

Your variables are:

```
term92kleft    term92klr    term92kright
```

```
>> term92kleft
```

```
term92kleft =
```

```
    91.5258    73.0261    41.6758    15.0544    % CASE: (b-=1,b+=0)
```

```
>> term92kright
```

```
term92kright =
```

```
    91.5258    73.0261    41.6758    15.0544    % CASE: (b-=0,b+=1)
```

```
>> term92klr
```

```
term92klr =
```

```
   183.0515   146.0523    83.3516    30.1088    % CASE: (b-=1,b+=1)
```

```
>>
```

```
%      ^           ^           ^           ^
%  I = 8      I = 9      I = 12      I = 20
```

```
%  -----
```

```
%      ^
%  VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #9      %
%  betatau = 2000 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term92k.out CREATED FROM THE  
 BINARY FILE term92k.mat USING MATLAB INTERACTIVE  
 COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
 USING A TEXT EDITOR.





```

        end
    end
    if ct == M(i)/2
        term10500left(i) = term10sum(i);
    end
    end
    term10500lr(i) = term10sum(i);
    term10500right(i) = term10500lr(i)-term10500left(i);
end
save term10500 term10500left term10500right term10500lr
exit

```

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 Commands for more information: help, whatsnew, info, subscribe

```
>> load term10500.mat
>> who
```

Your variables are:

```
term10500left      term10500right
term10500lr
```

```
>> term10500left
```

```
term10500left =
```

```
    -2.4375    -1.5672    -0.6099    -0.0927    % CASE: (b-=1,b+=0)
```

```
>> term10500right
```

```
term10500right =
```

```
    -2.4375    -1.5672    -0.6099    -0.0927    % CASE: (b-=0,b+=1)
```

```
>> term10500lr
```

```
term10500lr =
```

```
    -4.8750    -3.1343    -1.2197    -0.1854    % CASE: (b-=1,b+=1)
```

```
>>
```

```
%      ^      ^      ^      ^
%      I = 4      I = 5      I = 8      I = 20
%      -----
%      ^
```

```
% VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #10      %
%      betatau = 500  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term10500.out CREATED FROM THE BINARY FILE  
 term10500.mat USING MATLAB INTERACTIVE COMMANDS. ALL TEXT  
 PRECEDED BY A % WAS ADDED LATER USING A TEXT EDITOR.



```

        end
    end
    if ct == M(i)/2
        term101kleft(i) = term10sum(i);
    end
end
term101klr(i) = term10sum(i);
term101kright(i) = term101klr(i)-term101kleft(i);
end
save term101k term101kleft term101kright term101klr
exit

```

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 Version 4.1  
 Jun 15 1993

Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

```
>> load term101k.mat
>> who
```

Your variables are:

```
term101kleft      term101kright
term101klr
```

```
>> term101kleft
```

```
term101kleft =

    -0.0200    -0.0142    -0.0064    -0.0013    % CASE: (b-=1,b+=0)
```

```
>> term101kright
```

```
term101kright =

    -0.0200    -0.0142    -0.0064    -0.0013    % CASE: (b-=0,b+=1)
```

```
>> term101klr
```

```
term101klr =

    -0.0401    -0.0283    -0.0128    -0.0026    % CASE: (b-=1,b+=1)
```

```
>>
%      ^      ^      ^      ^
%      I = 5      I = 6      I = 9      I = 20
%      -----
%
% VALUES OF THE NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #10      %
% betatau = 1000 %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term101k.out CREATED FROM THE  
 BINARY FILE term101.mat USING MATLAB INTERACTIVE  
 COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
 USING A TEXT EDITOR.



```
        if ct == M(i)/2
            term1015kleft(i) = term10sum(i);
        end
    end
    term1015klr(i) = term10sum(i);
    term1015kright(i) = term1015klr(i)-term1015kleft(i);
end
save term1015k term1015kleft term1015kright term1015klr
exit
```



< M A T L A B (R) >  
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Commands to get started: intro, demo, help help  
 Commands for more information: help, whatsnew, info, subscribe

```
>> load term1015k.mat
>> who
```

Your variables are:

```
term1015kleft      term1015kright
term1015klr
```

```
>> term1015kleft
```

```
term1015kleft =
```

```
1.0e-04 *
```

```
-1.0000  -0.6199  -0.3539  -0.1279  % CASE: (b-=1,b+=0)
```

```
>> term1015kright
```

```
term1015kright =
```

```
1.0e-04 *
```

```
-1.0000  -0.6199  -0.3539  -0.1279  % CASE: (b-=0,b+=1)
```

```
>> term1015klr
```

```
term1015klr =
```

```
1.0e-03 *
```

```
-0.2000  -0.1240  -0.0708  -0.0256  % CASE: (b-=1,b+=1)
```

```
>>
```

```
%      ^      ^      ^      ^
%  I = 7      I = 9      I = 12     I = 20
```

```
%  -----
```

```
%      ^
%  VALUES OF NORMALIZED CHANNEL SPACING INTEGER I
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      TERM #10      %
%  betatau = 1500  %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

NOTE: THIS IS THE FILE term1015k.out CREATED FROM THE  
BINARY FILE term1015k.mat USING MATLAB INTERACTIVE  
COMMANDS. ALL TEXT PRECEDED BY A % WAS ADDED LATER  
USING A TEXT EDITOR.



```

        end
    end
    if ct == M(i)/2
        term102kleft(i) = term10sum(i);
    end
    end
    term102klr(i) = term10sum(i);
    term102kright(i) = term102klr(i)-term102kleft(i);
end
save term102k term102kleft term102kright term102klr
exit

```

```

>> term102kleft8
term102kleft8 =
    -6.5584e-07      % CASE: (b-=1,b+=0)

>> term102kright8
term102kright8 =
    -6.5584e-07      % CASE: (b-=0,b+=1)
                                % I = 8 %
                                % I = 8 %
                                % I = 8 %

>> term102klr8
term102klr8 =
    -1.3117e-06      % CASE: (b-=1,b+=1)

>>
-----
>> term102kleft9
term102kleft9 =
    -5.2745e-07      % CASE: (b-=1,b+=0)

>> term102kright9
term102kright9 =
    -5.2745e-07      % CASE: (b-=0,b+=1)
                                % I = 9 %
                                % I = 9 %
                                % I = 9 %

>> term102klr9
term102klr9 =
    -1.0549e-06      % CASE: (b-=1,b+=1)

>>
-----
>> term102kleft12
term102kleft12 =
    -3.0548e-07      % CASE: (b-=1,b+=0)

>> term102kright12
term102kright12 =
    -3.0548e-07      % CASE: (b-=0,b+=1)
                                % I = 12 %
                                % I = 12 %
                                % I = 12 %

```

>> term102klr12

term102klr12 =

-6.1096e-07           % CASE: (b-=1,b+=1)

>>

-----  
>> term102kleft20

term102kleft20 =

-1.1211e-07           % CASE: (b-=1,b+=0)

>> term102kright20

term102kright20 =

-1.1211e-07           % CASE: (b-=0,b+=1)

%%%%%%%%%%  
% I = 20 %  
%%%%%%%%%%

>> term102klr20

term102klr20 =

-2.2423e-07           % CASE: (b-=1,b+=1)

>>

-----  
%%%%%%%%%%  
%     TERM #10     %  
% betatau = 2000 %  
%%%%%%%%%%

NOTE: THERE WERE SEVERAL MISHAPS IN COMPUTING THIS TERM. POWER OUTAGES, MY ACCOUNT BEING SHUT OFF, ETC. IN THE INTERESTS OF BEING ABLE TO COMPUTE RESULTS IN A TIMELY MANNER, FOUR COPIES OF THE PROGRAM FOR TERM #10, betatau = 2000 WERE MADE. THE I AND M BLOCK WERE MODIFIED TO HOLD ONLY ONE VALUE OF I/M, AND THE OUTER LOOP WAS REDUCED TO ONE ITERATION. THEN, THE FOUR PROGRAMS WERE RUN ON FOUR SEPARATE WORKSTATIONS. IN THE INTERESTS OF BREVITY WE ONLY SHOW THE SINGLE PROGRAM FOR COMPUTING THE TERM AS THE FOUR COPIES ARE ESSENTIALLY THE EXACT SAME PROGRAM BUT THEY DO ONLY ONE OUTER LOOP EACH. THE RESULTS OF THE FOUR PROGRAMS ARE TABULATED HERE USING THE A TEXT EDITOR.



```

else
    X5 = 0;
end
X6 = boo*bitmatrix(i,3)*c*1.8585e+03;
X7 = boo*bitmatrix(i,3)*c*9.2657e+01;
if boo == 0
    X8 = 0;
elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
    X8 = c*term8(1,ct);
elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
    X8 = c*term8(2,ct);
elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
    X8 = c*term8(3,ct);
else
    X8 = 0;
end
if bitmatrix(i,3) == 0
    X9 = 0;
elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
    X9 = c*term9(1,ct);
elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
    X9 = c*term9(2,ct);
elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
    X9 = c*term9(3,ct);
else
    X9 = 0;
end
if bitmatrix(i,3) ==0
    X10 = 0;
elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
    X10 = c*term10(1,ct);
elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
    X10 = c*term10(2,ct);
elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
    X10 = c*term10(3,ct);
else
    X10 = 0;
end
INTERSUM = X1+X2+X3+X4+X5+X6+X7+X8+X9+X10;
if boo==1
    X1BETA(ct,i) = INTERSUM;
else
    XOBETA(ct,i) = INTERSUM;
end
end
end
X1MIN(ct) = min(X1BETA(ct,:));
XOMAX(ct) = max(XOBETA(ct,:));
VT(ct) = (XOMAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;

```



```

RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
    PE(ct,1:1001) = zeros(1,1001);

    for i = 1:8
        PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
            +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-X0BETA(ct,i)));
    end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
    RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
    RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])

```



```

        if boo == 0
            X8 = 0;
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X8 = c*term8(1,ct);
        elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X8 = c*term8(2,ct);
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X8 = c*term8(3,ct);
        else
            X8 = 0;
        end
        if bitmatrix(i,3) == 0
            X9 = 0;
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X9 = c*term9(1,ct);
        elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X9 = c*term9(2,ct);
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X9 = c*term9(3,ct);
        else
            X9 = 0;
        end
        if bitmatrix(i,3) ==0
            X10 = 0;
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X10 = c*term10(1,ct);
        elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X10 = c*term10(2,ct);
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X10 = c*term10(3,ct);
        else
            X10 = 0;
        end
        INTERSUM = X1+X2+X3+X4+X6+X7+X8+X9+X10;
        if boo==1
            X1BETA(ct,i) = INTERSUM;
        else
            XOBETA(ct,i) = INTERSUM;
        end
    end
end
X1MIN(ct) = min(X1BETA(ct,:));
XOMAX(ct) = max(XOBETA(ct,:));
VT(ct) = (XOMAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
    PE(ct,1:1001) = zeros(1,1001);
    for i = 1:8

```

```

        PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
            +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-X0BETA(ct,i)));
    end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
    RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
    RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])

```



```

        X8 = 0;
    elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
        X8 = c*term8(1,ct);
    elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
        X8 = c*term8(2,ct);
    elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
        X8 = c*term8(3,ct);
    else
        X8 = 0;
    end
    if bitmatrix(i,3) == 0
        X9 = 0;
    elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
        X9 = c*term9(1,ct);
    elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
        X9 = c*term9(2,ct);
    elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
        X9 = c*term9(3,ct);
    else
        X9 = 0;
    end
    if bitmatrix(i,3) ==0
        X10 = 0;
    elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
        X10 = c*term10(1,ct);
    elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
        X10 = c*term10(2,ct);
    elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
        X10 = c*term10(3,ct);
    else
        X10 = 0;
    end
    INTERSUM = X1+X2+X3+X4+X6+X7+X8+X9+X10;
    if boo==1
        X1BETA(ct,i) = INTERSUM;
    else
        XOBETA(ct,i) = INTERSUM;
    end
end
end
end
X1MIN(ct) = min(X1BETA(ct,:));
XOMAX(ct) = max(XOBETA(ct,:));
VT(ct) = (XOMAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
    PE(ct,1:1001) = zeros(1,1001);
    for i = 1:8
        PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...

```

```

                                +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-XOBETA(ct,i)));
    end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
          RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
          RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])

```





```

        if boo == 0
            X8 = 0;
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X8 = c*term8(1,ct);
        elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X8 = c*term8(2,ct);
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X8 = c*term8(3,ct);
        else
            X8 = 0;
        end
        if bitmatrix(i,3) == 0
            X9 = 0;
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X9 = c*term9(1,ct);
        elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X9 = c*term9(2,ct);
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X9 = c*term9(3,ct);
        else
            X9 = 0;
        end
        if bitmatrix(i,3) ==0
            X10 = 0;
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==0
            X10 = c*term10(1,ct);
        elseif bitmatrix(i,1)==0 & bitmatrix(i,2)==1
            X10 = c*term10(2,ct);
        elseif bitmatrix(i,1)==1 & bitmatrix(i,2)==1
            X10 = c*term10(3,ct);
        else
            X10 = 0;
        end
        INTERSUM = X1+X2+X3+X4+X6+X7+X8+X9+X10;
        if boo==1
            X1BETA(ct,i) = INTERSUM;
        else
            XOBETA(ct,i) = INTERSUM;
        end
    end
end
X1MIN(ct) = min(X1BETA(ct,:));
XOMAX(ct) = max(XOBETA(ct,:));
VT(ct) = (XOMAX(ct) + X1MIN(ct))/2;
end
RPSQR_TDIVNO_DB = 10:.01:20;
RPSQR_TDIVNO = 10.^(RPSQR_TDIVNO_DB*0.10);
SINGCHAN = 0.5*erfc(RPSQR_TDIVNO/8^0.5);
for ct = 1:4
    PE(ct,1:1001) = zeros(1,1001);
    for i = 1:8

```

```

        PE(ct,:)=PE(ct,:)+0.25*erfc((RPSQR_TDIVNO/2^0.5)*(X1BETA(ct,i)-VT(ct)))...
            +0.25*erfc((RPSQR_TDIVNO/2^0.5)*(VT(ct)-X0BETA(ct,i)));
    end
end
PEFINAL = PE/8;
figure(1)
semilogy(RPSQR_TDIVNO_DB,SINGCHAN,'--',RPSQR_TDIVNO_DB,PEFINAL(1,:),...
    RPSQR_TDIVNO_DB,PEFINAL(2,:),RPSQR_TDIVNO_DB,PEFINAL(3,:),...
    RPSQR_TDIVNO_DB,PEFINAL(4,:))
xlabel('Z (dB)');
ylabel('Pb');
axis([10 17 10^(-15) 1])

```

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